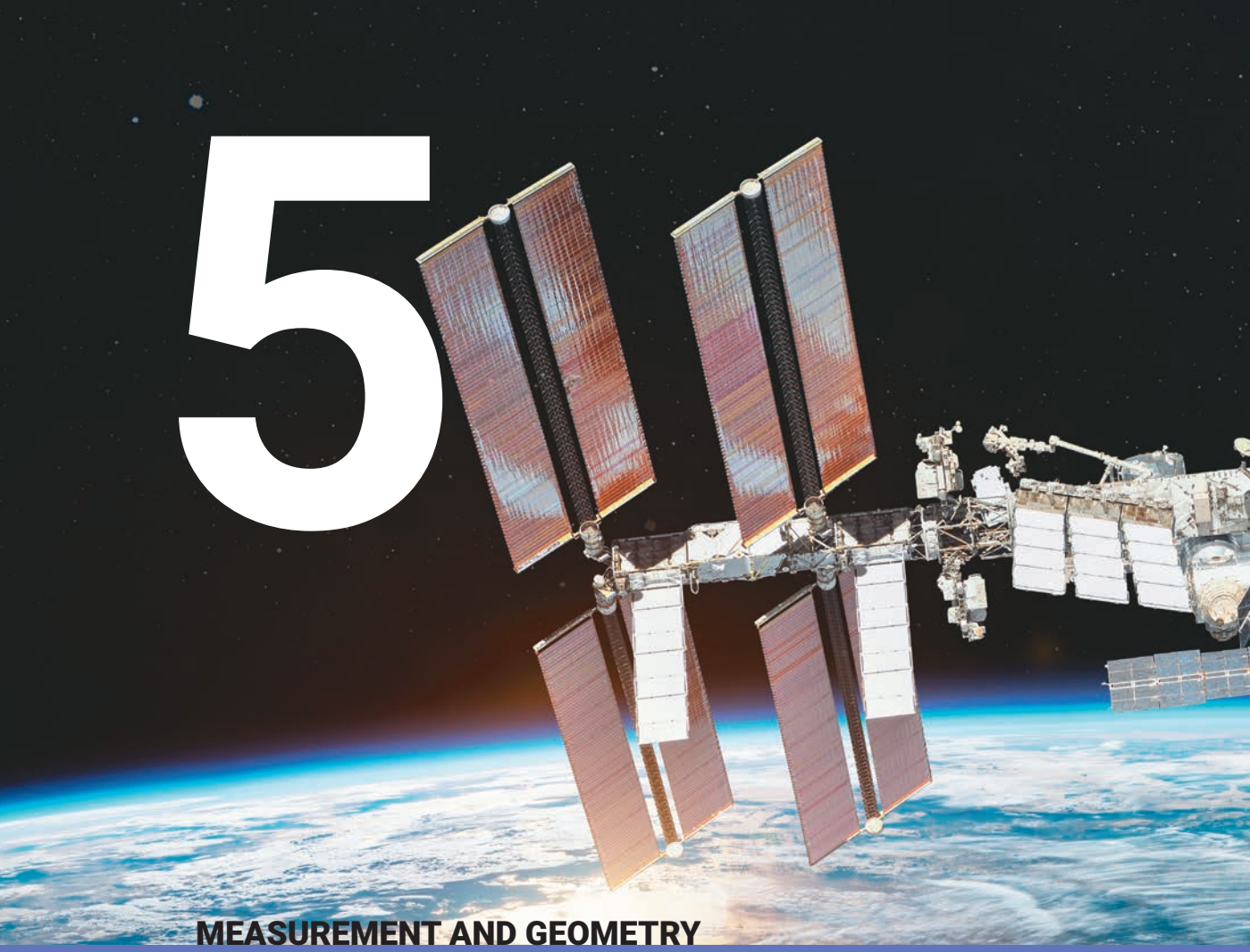
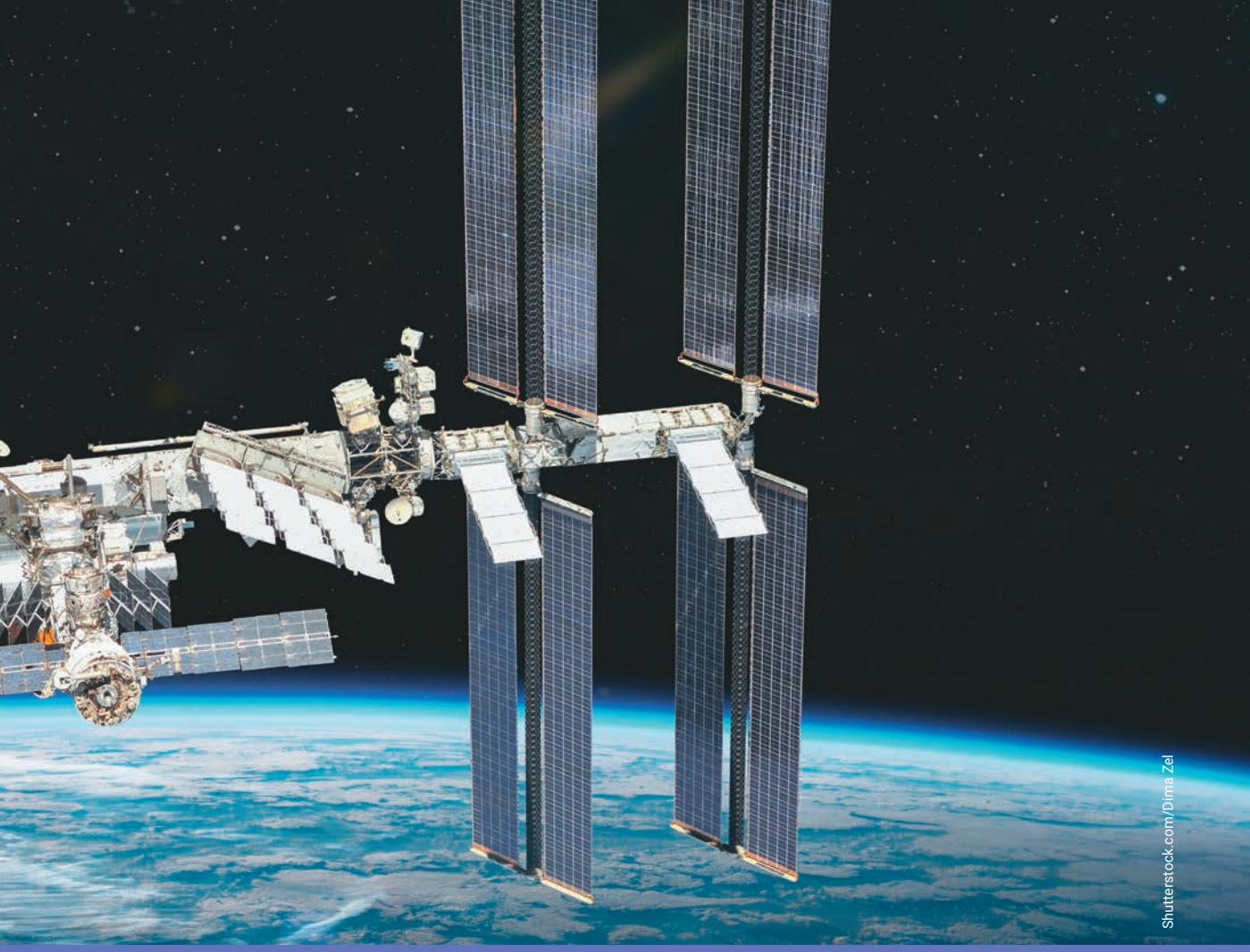


5

MEASUREMENT AND GEOMETRY

AREA AND VOLUME

The International Space Station (ISS) is the largest man-made object in space.
The construction of the ISS began in 1998. It circles the Earth every 90 minutes at a height of approximately 400 km above the Earth. ISS has a pressurised volume of 915 m^3 and it is 109 metres long, the same length as a Boeing 707. The solar panels of the station cover an area of 2500 m^2 and produce enough electricity to power 10 average-sized homes.



Chapter outline

	Working mathematically				
5.01 Perimeter	U	F	PS	R	
5.02 Metric units for area	U	F		R	C
5.03 Areas of rectangles, triangles and parallelograms	U	F	PS	R	
5.04 Areas of composite shapes	U	F	PS	R	
5.05 Area of a trapezium	U	F	PS	R	
5.06 Areas of kites and rhombuses	U	F			
5.07 Parts of a circle	U	F		R	C
5.08 Circumference of a circle	U	F	PS	R	
5.09 Area of a circle	U	F	PS	R	
5.10 Perimeter and area of a sector	U	F	PS	R	
5.11 Metric units for volume	U	F		R	C
5.12 Volume of a prism	U	F	PS	R	C
5.13 Volume of a cylinder	U	F	PS	R	
5.14 Volume and capacity	U	F	PS	R	C

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Wordbank

capacity The amount of fluid (liquid or gas) in a container

circumference The perimeter of a circle; a circle's outer boundary

cross-section A 'slice' of a solid, taken across the solid rather than along it

cubic metre The volume of a cube that measures 1 m by 1 m by 1 m

perpendicular height The height of a shape taken at right angles to its base

pi (π) A special irrational number, approximately 3.1416, used in calculating circular measurements

radius The distance from the centre of a circle to the circle's edge

sector A 'pizza-slice' of a circle, cut along 2 radii

In this chapter you will:

- calculate perimeters and areas of triangles, quadrilaterals and composite shapes
- convert between units of area
- learn the terminology for parts of a circle
- investigate the line and rotational symmetries of circles and circular shapes
- calculate perimeters and areas of circles, sectors and circular shapes, giving answers in terms of π and as decimals
- convert between units of area
- convert between units of area and capacity
- calculate volumes of prisms and cylinders

Multiplying by 10, 100, 1000

SkillCheck ANSWERS ON P. 557

1 Copy and complete each conversion.

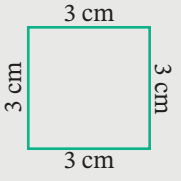
a 20 cm = _____ mm **b** 350 cm = _____ m **c** 2500 m = _____ km

d 46 mm = _____ cm **e** 4 km = _____ m **f** 5200 m = _____ km

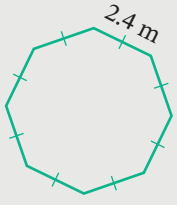
g 600 m = _____ km **h** 8200 mm = _____ m **i** 0.4 m = _____ mm

2 Find the perimeter of each shape.

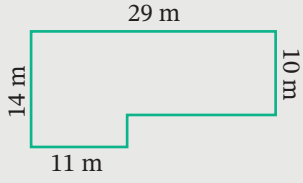
a



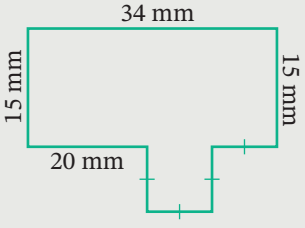
b



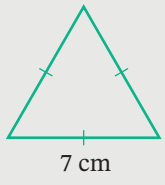
c



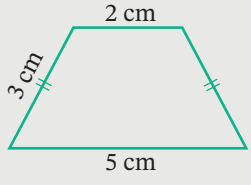
d



e

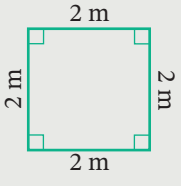


f



3 Find the area of each shape.

a



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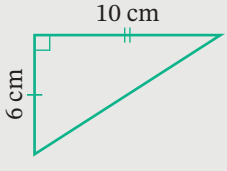

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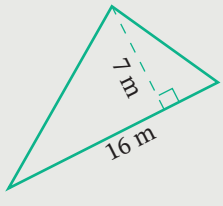

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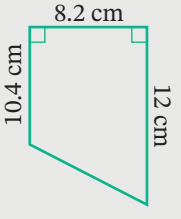

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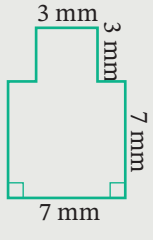

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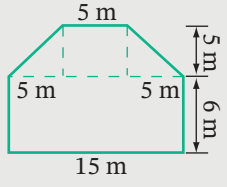

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Perimeter 5.01

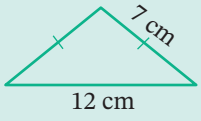
Perimeter

The **perimeter** of a shape is the distance around the shape.
It is the sum of the lengths of the sides of the shape.

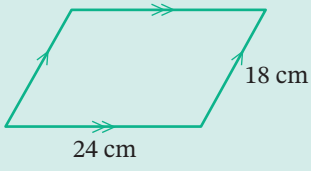
Example 1

Find the perimeter of each shape.

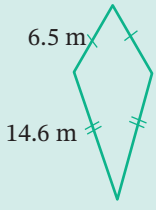
a



b



c



Solution

a Perimeter $= 7 + 7 + 12$
$= 26$ cm

Adding the 3 sides of the isosceles triangle.

b Perimeter $= 24 + 18 + 24 + 18$
$= 84$ cm

In a parallelogram, opposite sides are equal.

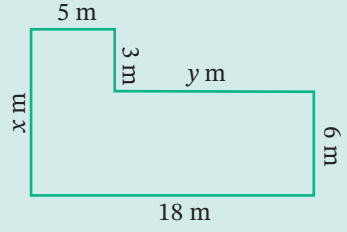
c Perimeter $= 6.5 + 6.5 + 14.6 + 14.6$
$= 42.2$ m

In a kite, adjacent sides are equal.

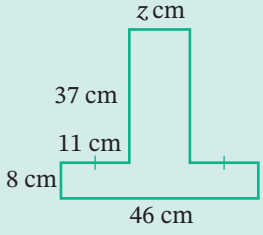
Example 2

Find the perimeter of each composite shape.

a



b



Solution

a Find the unknown sides first.
$x = 3 + 6 = 9$
$y = 18 - 5 = 13$

Perimeter $= 9 + 5 + 3 + 13 + 6 + 18$
$= 54$ m

b Find the unknown sides first.
$z = 46 - 11 - 11 = 24$

Perimeter $= 8 + 11 + 37 + 24 + 37 + 11 + 8 + 46$
$= 182$ cm

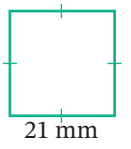
EXERCISE 5.01 ANSWERS ON P. 557

Perimeter U F PS R

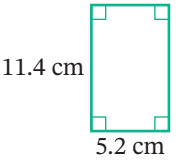
EXAMPLE 1

1 Find the perimeter of each shape.

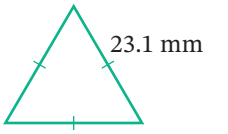
a



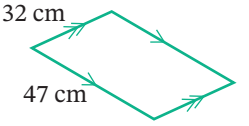
b



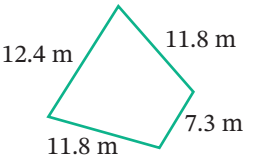
c



d



e



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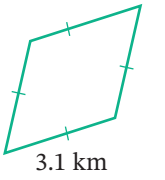
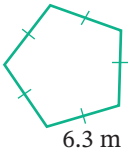


EXAMPLE 2

2 Find the perimeter of each shape.

a



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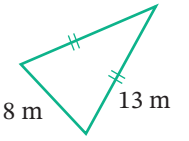

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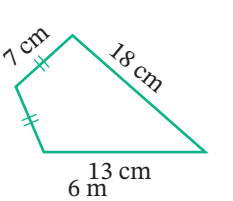

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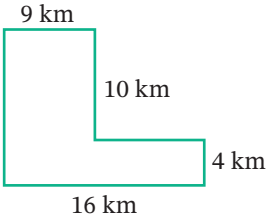
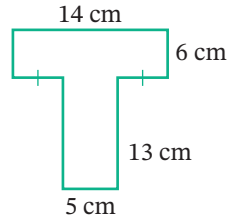


e



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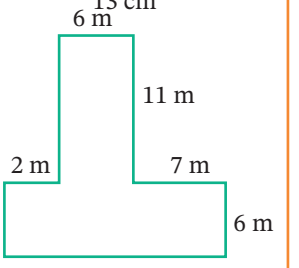
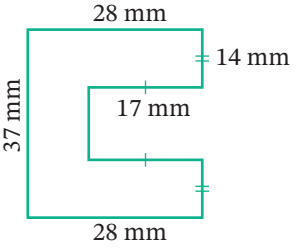
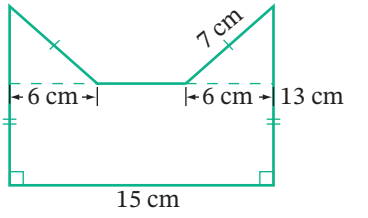


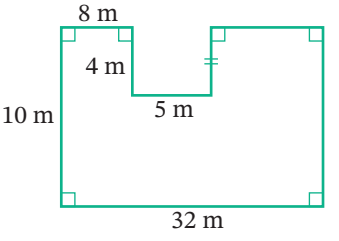
g



h



i



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3 Find the perimeter of:

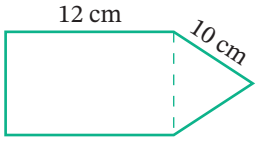
a a square of side length 3 cm

b a parallelogram with adjacent sides of length 7 m and 10 m

c an equilateral triangle of side length 5 cm

d an isosceles triangle with equal sides of 9 m and a side of 4 m

4 This composite shape is made up of a rectangle and an equilateral triangle. Find its perimeter.
Select the correct answer **A**, **B**, **C** or **D**.



A 22 cm **B** 44 cm

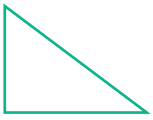
C 54 cm **D** 64 cm

5 Which of the following lengths is the closest to the perimeter of your classroom?
Select **A**, **B**, **C** or **D**. R

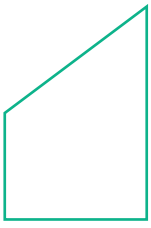
A 250 m **B** 2.5 km **C** 250 cm **D** 25 m

6 Measure the sides of each shape in millimetres and find its perimeter.

a



b



c

d

7 A square has a perimeter of 24 cm. What is the length of one side?
Select **A**, **B**, **C** or **D**. R

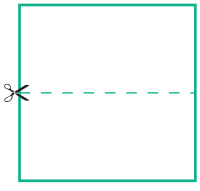
A 6 cm **B** 8 cm **C** 12 cm **D** 96 cm

8 A rectangle has a perimeter of 40 m. If its length is 13 m, find its width. R

9 A rhombus has a perimeter of 18 m. Find the length of one side. R

10 A kite has a perimeter of 26 cm. If one of its longer sides is 9 cm, find the length of a shorter side. R

11 A square is cut in half as shown. The 2 rectangles have a combined perimeter of 24 cm. Find the perimeter of the original square. PS R



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12 Jade uses a piece of wire to build a square with a side of 12 cm. If, from the same wire, she builds a rectangle with a width of 6 cm, what is the length of the rectangle? PS R

13 Farouk's rectangular garden is 14 metres long and 10 metres wide. A hose is buried inside the garden $1\frac{1}{2}$ metres from each side. How long is the hose? PS R

5.02 Metric units for area

What is area?

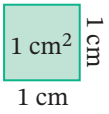
Australian areas

The **area** of a shape is the amount of surface covered by the shape. Area is measured in **square units**.

square centimetre (cm^2)

1 cm = 10 mm

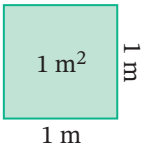
$1\ cm^2 = 1\ cm \times 1\ cm$
$= 10\ mm \times 10\ mm$
$= 100\ mm^2$



square metre (m^2)

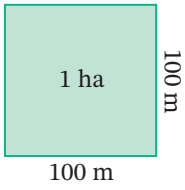
1 m = 100 cm

$1\ m^2 = 1\ m \times 1\ m$
$= 100\ cm \times 100\ cm$
$= 10000\ cm^2$



hectare (ha)

$1\ ha = 100\ m \times 100\ m$
$= 10000\ m^2$

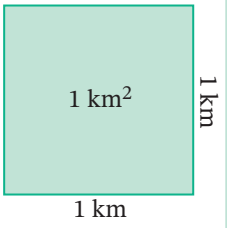


square kilometre (km^2)

$1\ km = 1000\ m$

$1\ km^2 = 1\ km \times 1\ km$
$= 1000\ m \times 1000\ m$
$= 1000\ 000\ m^2$

$1\ km^2 = 100\ ha$



A **square centimetre** is about the area of a fingernail.

A **square metre** is approximately the area of the floor of a large shower recess.

A **hectare** is about the area of Sydney Football Stadium, or the grassed area inside an athletics (400 m) track.

A **square kilometre** is about the area of a theme park, like Dreamworld on the Gold Coast.

Note that, while 1 cm = 10 mm, $1\ cm^2 = 100\ mm^2$ (double the number of 0s), and while 1 m = 100 cm, $1\ m^2 = 10\ 000\ cm^2$ (double the number of 0s).

Metric units of area

$1\ cm^2 = 100\ mm^2$

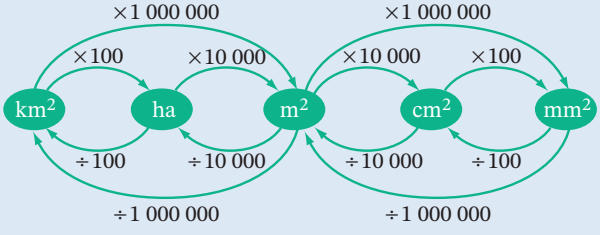
$1\ m^2 = 10\ 000\ cm^2$

$1\ m^2 = 1\ 000\ 000\ mm^2$

$1\ ha = 10\ 000\ m^2$

$1\ km^2 = 1\ 000\ 000\ m^2$

$1\ km^2 = 100\ ha$



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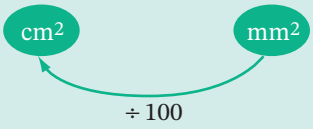
Example 3

Convert:

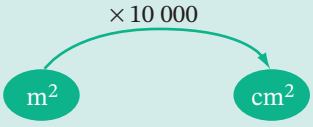
a 800 mm^2 to cm^2 **b** 4.6 m^2 to cm^2 **c** 25 000 000 m^2 to ha

Solution

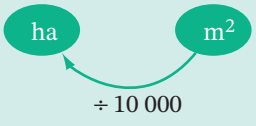
a $800\ mm^2 = 800 \div 100\ cm^2$
$= 8\ cm^2$



b $4.6\ m^2 = 4.6 \times 10\,000\ cm^2$
$= 46\,000\ cm^2$



c $25\,000\,000\ m^2 = 25\,000\,000 \div 10\,000\ cm^2$
$= 2500$ ha



EXERCISE 5.02 ANSWERS ON P. 557

Metric units for area U F R C

1 What units of area would you use when measuring the area of: R C

a your home? **b** your pencil case? **c** a football field?
d the state of NSW? **e** your body? **f** a desk top?

2 How many square centimetres are there in 3.57 m^2?
Select the correct answer **A**, **B**, **C** or **D**. R

A 35 700 **B** 357 **C** 0.357 **D** 0.0357

3 Copy and complete each conversion. R

a 100 mm^2 = _____ cm^2 **b** 750 mm^2 = _____ cm^2
c 4500 cm^2 = _____ m^2 **d** 34 000 m^2 = _____ km^2
e 60 000 mm^2 = _____ m^2 **f** 432 000 cm^2 = _____ m^2
g 950 mm^2 = _____ m^2 **h** 7600 m^2 = _____ ha
i 4.8 mm^2 = _____ cm^2 **j** 30 m^2 = _____ cm^2
k 850 cm^2 = _____ mm^2 **l** 6.9 km^2 = _____ m^2
m 54 cm^2 = _____ mm^2 **n** 700 ha = _____ m^2
o 0.48 m^2 = _____ cm^2 **p** 0.45 km^2 = _____ m^2

4 How many square metres are there in one square kilometre? Select **A**, **B**, **C** or **D**. R

A 1000 **B** 10 000 **C** 100 000 **D** 1 000 000

EXAMPLE 3

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5 The solar panels of the International Space Station cover an area of 2500 m^2. What is this area in km^2? R C

6 A large cattle station in South Australia has an area of 30 028 km^2. What is this in hectares? R C

7 The Cradle Mountain–St Clair National Park in Tasmania has an area of 134 805 ha. What is this in km^2? R C

8 The centre of Sydney is approximately a rectangle bounded by George Street, Circular Quay, Macquarie and College Streets and Liverpool Street. This rectangle is about 1.75 km by 0.5 km. What is its area in: R C

a km^2?

b hectares?

9 Copy and complete each conversion. R

a 7600 mm^2 = _____ m^2

b 0.7 ha = _____ cm^2

c 8 m^2 = _____ mm^2

d 850 mm^2 = _____ cm^2

e 6.90 ha = _____ m^2

f 15.2 m^2 = _____ cm^2

g 8500 m^2 = _____ ha

h 49 000 000 m^2 = _____ km^2

10 The area of Australia is 7 682 300 km^2. Western Australia has an area of 2 526 000 km^2. Calculate, correct to one decimal place, the area of Western Australia as a percentage of the area of Australia.

11 Sydney Airport has an area of 881 hectares. Sydney Harbour has an area of about 5500 hectares. R C

a What is the area of Sydney Airport in square kilometres?

b How many 'Sydney Airports' would fit onto Sydney Harbour? Answer correct to one decimal place.

5.03 Areas of rectangles, triangles and parallelograms

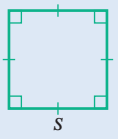
What is area?

WS Rectangle and triangle areas

Area formulas for triangles and quadrilaterals

Area formulas

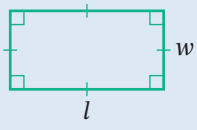
Square



$A = (\text{side})^2$

$A = s^2$

Rectangle



$A = \text{length} \times \text{width}$

$A = lw$

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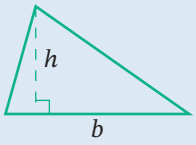
Area group clues

5.03

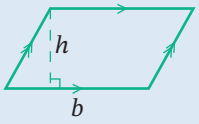
Triangle



$A = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$

$A = \frac{1}{2}bh$

Parallelogram



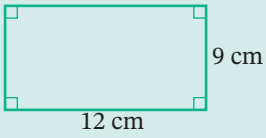
$A = \text{base} \times \text{perpendicular height}$

$A = bh$

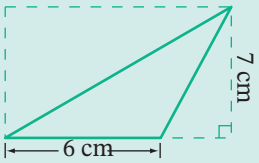
Example 4

Find the area of each shape.

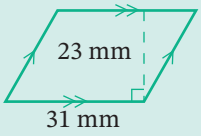
a



b



c



Solution

a $A = lw$

$A = 12 \times 9$

$= 108 \text{ cm}^2$

Area of a rectangle = length × width

b $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 7$

$= 21 \text{ cm}^2$

Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

c $A = bh$

$= 31 \times 23$

$= 713 \text{ m}^2$

Area of a parallelogram = base × height

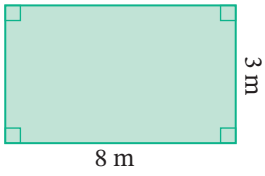
EXERCISE 5.03 ANSWERS ON P. 557

Areas of rectangles, triangles and parallelograms U F PS R

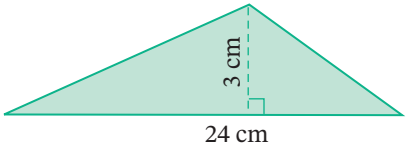
EXAMPLE 4

1 Find the area of each shape.

a



b



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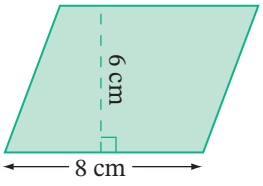
c



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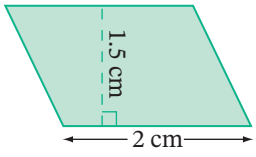

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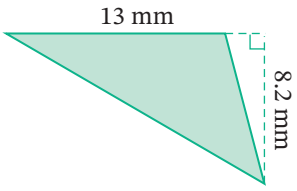

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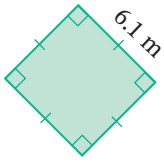

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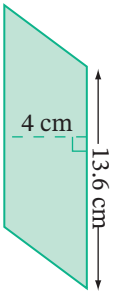

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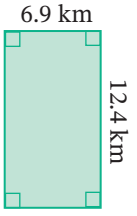

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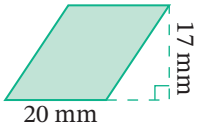

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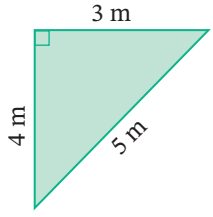
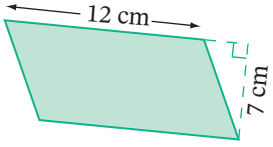
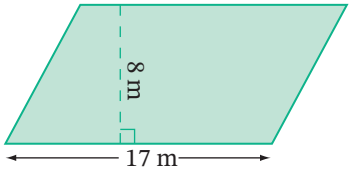


k

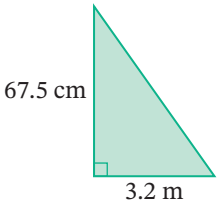


l

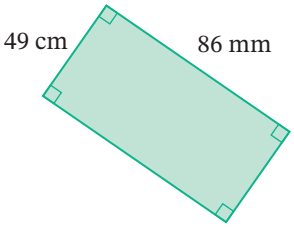


2 Find the area of each shape. Make sure that you change all lengths to the same units first.

a



b



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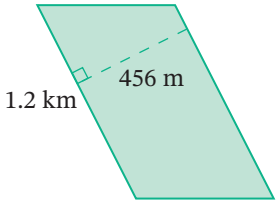

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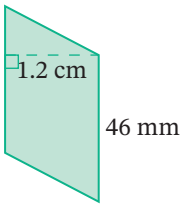
d



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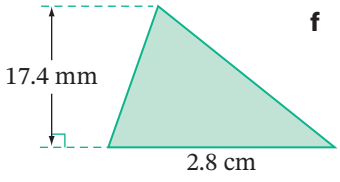

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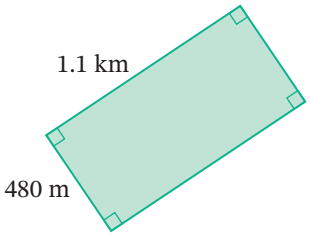
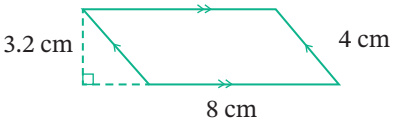


3 One hectare of land is subdivided into blocks. If the average block measures 20 metres by 25 metres, how many blocks would there be?
Select the correct answer **A**, **B**, **C** or **D**. PS R

A 50 **B** 25 **C** 40 **D** 20

4 Find the area of this shape. Select **A**, **B**, **C** or **D**.



A 12.8 cm^2 **B** 24 cm^2

C 25.6 cm^2 **D** 32 cm^2

5 A square has an area of 25 cm^2. Find the length of one side. R

6 A rectangle has an area of 27 m^2. If its length is 9 m, find its width. R

7 A triangle has an area of 35 cm^2. If its base length is 10 cm, what is its perpendicular height? R

8 A rectangle has an area of 72 mm^2. What could its length and width be? R

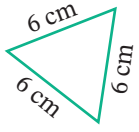
9 The area of a square is 900 m^2. If the area of the square quadrupled, what would be the length of the new square? PS R

10 The area of a triangle can be found using **Heron's formula** if you know the lengths of all 3 sides a, b and c:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \qquad \text{where } s = \text{half the perimeter} = \frac{a+b+c}{2}$$

Find the area of each triangle using Heron's formula, correct to one decimal place.

a



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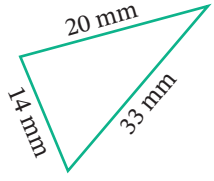
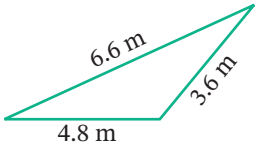


c



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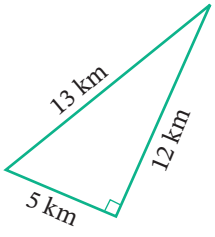

5.03

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5.04 Areas of composite shapes

Areas of composite shapes

A page of composite shapes

Odd areas

Areas 1

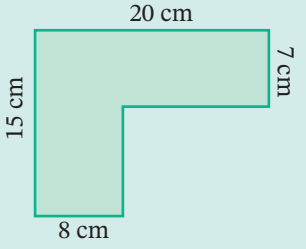
Area puzzles

Shapes made by combining other simpler shapes are called **composite shapes.**

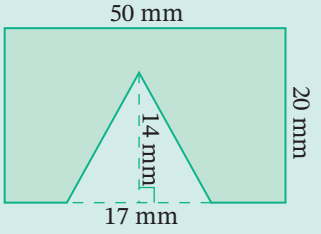
Example 5

Find the area of each composite shape.

a



b



Solution

a **Method 1**

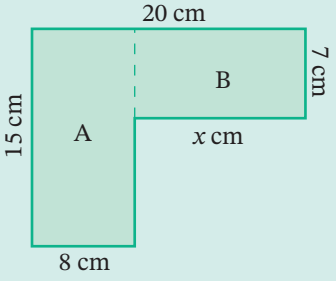
Divide the shape into 2 rectangles, A and B, and find any unknown lengths.

$x = 20 - 8 = 12$

Area of shape = area of A + area of B

$= 8 \times 15 + 12 \times 7$

$= 204 \text{ cm}^2$



Method 2

The area can also be found by subtracting areas.

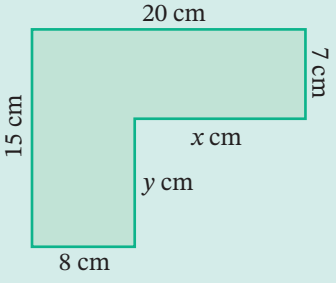
$x = 20 - 8 = 12$

$y = 15 - 7 = 8$

Area = large rectangle − small rectangle

$= 20 \times 15 - 12 \times 8$

$= 204 \text{ cm}^2$



b Area = area of rectangle − area of triangle

$= 50 \times 20 - \frac{1}{2} \times 17 \times 14$

$= 1000 - 119$

$= 881 \text{ mm}^2$

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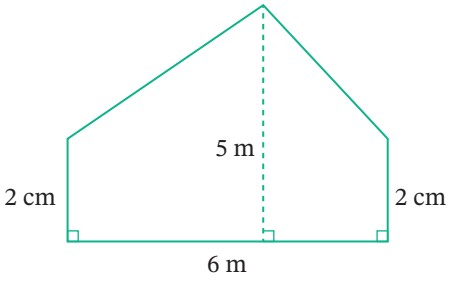
EXERCISE 5.04 ANSWERS ON P. 557

Areas of composite shapes U F PS R

1 What is the area of this shape? Select the correct answer **A**, **B**, **C** or **D**. R

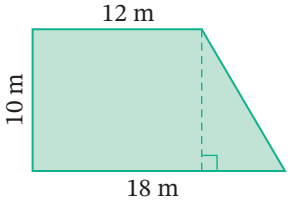
A 21 m² **B** 27 m²

C 30 m² **D** 42 m²

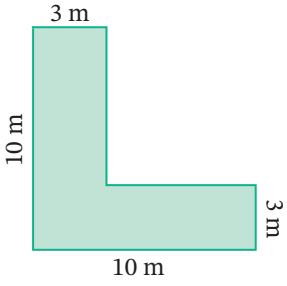


2 Find the area of each composite shape. PS R

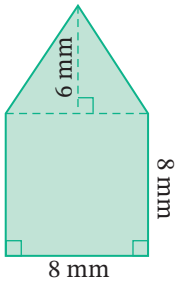
a



b



c



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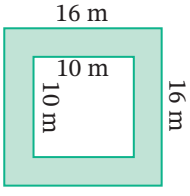

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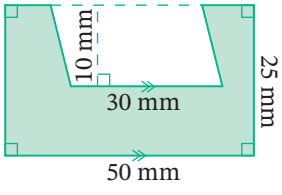

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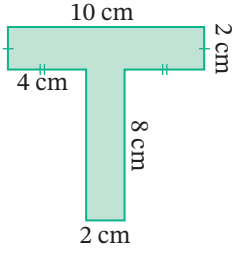

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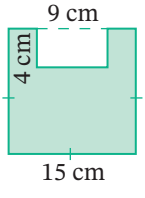

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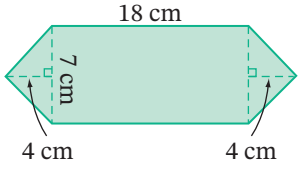

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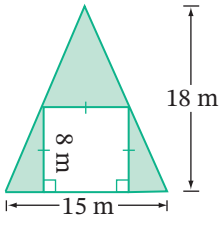

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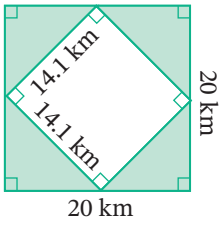

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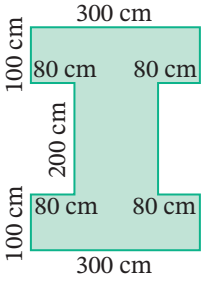
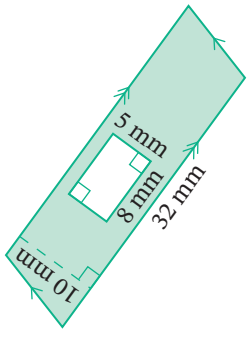
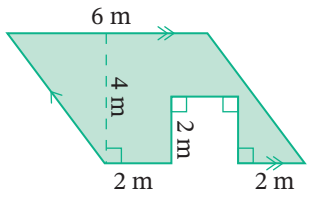


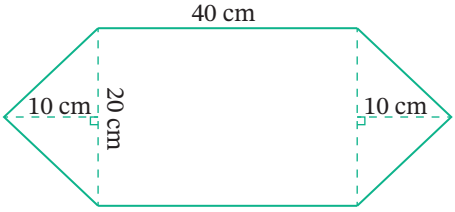
l



m



3 What is the area of this shape?
Select **A**, **B**, **C** or **D**.



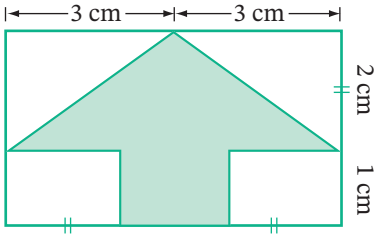
A 820 cm^2

B 900 cm^2

C 1000 cm^2

D 1200 cm^2

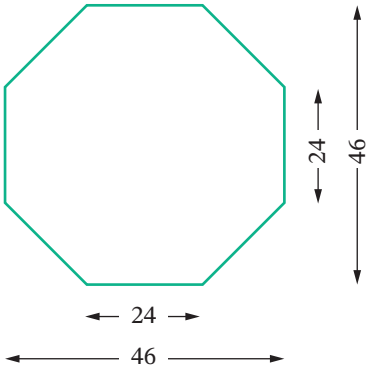
4 What is the area of the shaded region?
Select **A**, **B**, **C** or **D**. PS R



A 14 cm^2 **B** 8 cm^2

C 9 cm^2 **D** 12 cm^2

5 Find the area of this octagon. All dimensions in centimetres. PS R



Mental skills 5A: Maths without calculators ANSWERS ON P. 557

Converting fractions and percentages to decimals

To convert a fraction into a decimal, change the denominator to a power of 10. This may require simplifying the fraction first.

1 Study each example.

a $\frac{31}{50} = \frac{31 \times 2}{50 \times 2} = \frac{62}{100} = 0.62$

b $\frac{18}{40} = \frac{18 \div 2}{40 \div 2} = \frac{9}{20} = \frac{9 \times 5}{20 \times 5} = \frac{45}{100} = 0.45.$

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2 Now convert each fraction into a decimal.

a $\frac{7}{10}$ **b** $\frac{15}{25}$ **c** $\frac{14}{200}$

d $\frac{9}{20}$ **e** $\frac{49}{50}$ **f** $\frac{12}{30}$

To convert a percentage into a decimal, divide by 100 by moving the decimal point 2 places to the left.

3 Study each example.

a $18\% = 0.18$ **b** $70\% = 0.70 = 0.7$ **c** $11.4\% = 0.114$ **d** $2.9\% = 0.029$

4 Now convert each percentage to a decimal.

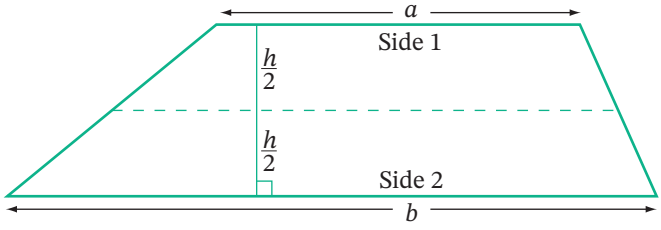
a 61% **b** 4% **c** 90%

d 38% **e** 27.1% **f** 0.7%

Investigation

Area of a trapezium

1 Draw 2 copies of this trapezium onto a sheet of paper. Draw a dotted line, parallel to the sides labelled a and b, halfway down the height of each trapezium and label the sides as shown.



2 Cut out one trapezium and then cut along the dotted line to make 2 smaller trapeziums. Join the pieces to make a long parallelogram.

3 By measuring the height and base of the parallelogram to the nearest millimetre, find its area.

4 By comparing the measurements of the parallelogram to the measurements of the original trapezium, suggest a general formula for finding the area of any trapezium. Check your answer with your teacher.

5 Cut out the original trapezium and paste it and the parallelogram into your book.

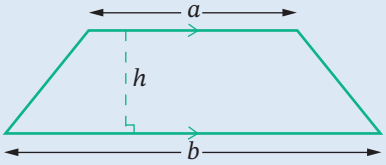
5.05 Area of a trapezium

Area formulas for triangles and quadrilaterals

Area of a trapezium

$A = \frac{1}{2} \times$ sum of lengths of parallel sides $\times$ perpendicular height

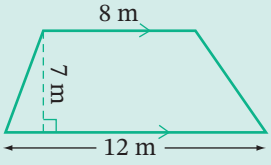
$A = \frac{1}{2}(a+b)h$



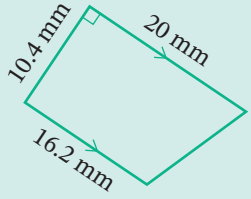
Example 6

Find the area of each trapezium.

a



b



Solution

a $A = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(8+12) \times 7$

$= 70\text{ m}^2$

b $A = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(20+16.2) \times 10.4$

$= 188.24\text{ mm}^2$

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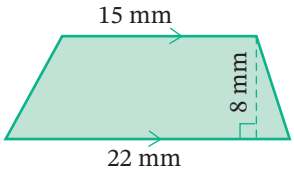
Area of a trapezium U F PS R

EXAMPLE 6

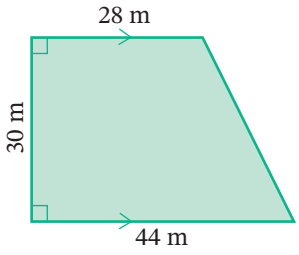
5.05

1 Find the area of each trapezium.

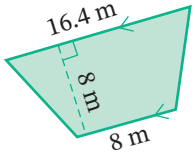
a



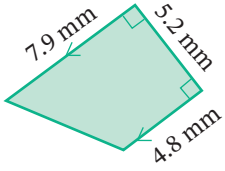
b



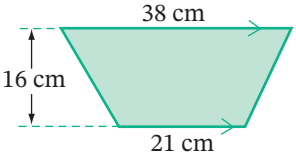
c



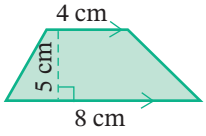
d



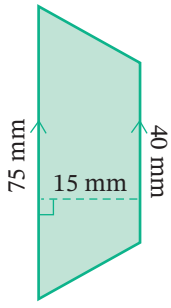
e



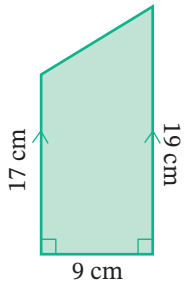
f



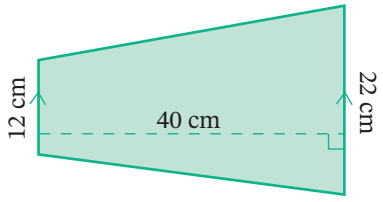
g



h

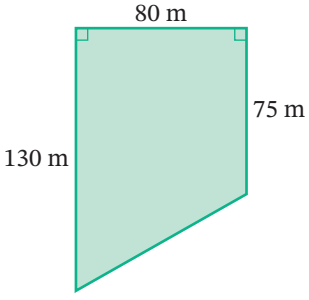


i

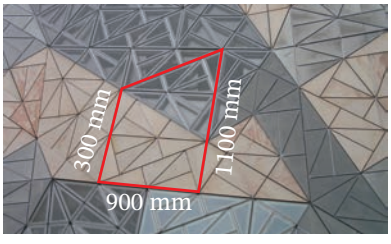


2 **a** Find the area of the block of land shown.

b Council regulations state that $\frac{1}{5}$ of the land must be reserved for gardens. How much land is available for building?



3 Find the area of the trapezoidal shape in the photo of the exterior of Federation Square, Melbourne.



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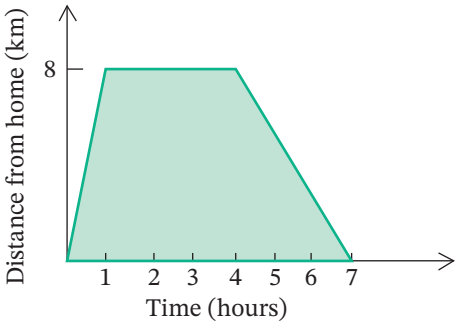
4 The travel graph shows a trip made by Katerina. What is the shaded area in the graph? Select **A**, **B**, **C** or **D**.

A 44 units^2 **B** 48 units^2

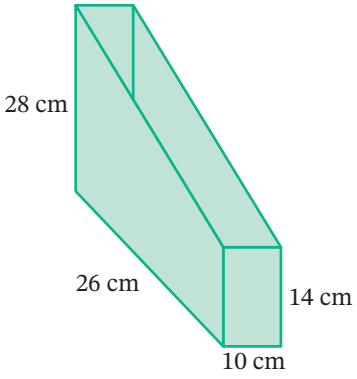
C 42 units^2 **D** 40 units^2



5 A magazine holder has trapezoidal sides as shown. PS R

a Find the area of one of the trapezoidal sides.

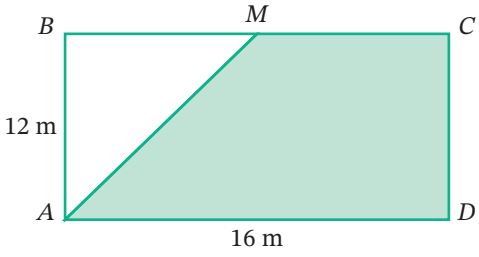
b Find the total area of cardboard needed to build this holder.



6 In the diagram, M is halfway between B and C. Calculate the shaded area. Select **A**, **B**, **C** or **D**. R

A 96 m^2 **B** 132 m^2

C 224 m^2 **D** 144 m^2



7 One side of the stage where Greta's drama club performs is shaped like a trapezium with an area of 144 m^2. The trapezium's parallel sides are 30 m and 42 m. What is the height of the trapezium? PS R

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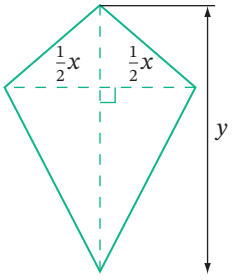
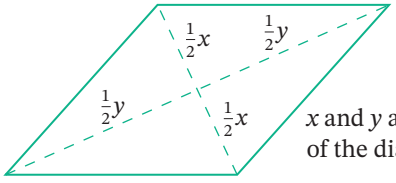
Investigation

Area of a kite and a rhombus

1 Draw 2 copies of the kite and rhombus shown below on a sheet of paper. Draw the diagonals of length x and y and label them as shown. Notice that they cross at right angles. For the kite, one diagonal (x) is bisected by the other (y), while for the rhombus, *both* diagonals bisect each other.





x and y are the lengths of the diagonals

2 Cut out one of your kites and rhombuses and cut along their diagonals to make 4 triangles each. Join each set of triangles to make a rectangle.

3 By comparing the measurements of the kite to the measurements of the rectangle formed from it, suggest a general formula for finding the area of any kite. Check your answer with your teacher.

4 **a** What is the definition of a kite?

b Is a rhombus a special type of kite?

5 By comparing the measurements of the rhombus to the measurements of the rectangle formed from it, suggest a general formula for finding the area of any rhombus. Check your answer with your teacher.

6 Cut out the original kite and rhombus and paste all shapes into your book.

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5.06 Areas of kites and rhombuses

Area formulas for triangles and quadrilaterals

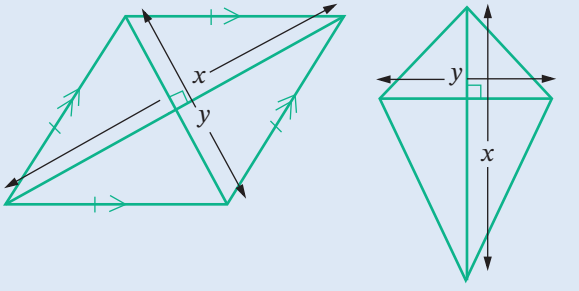
WS Quadrilateral and triangle areas

Areas of quadrilaterals and triangles

Area of a kite or a rhombus

$A = \frac{1}{2} \times (\text{diagonal}) \times (\text{other diagonal})$

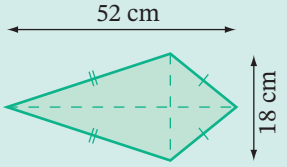
$A = \frac{1}{2}xy$



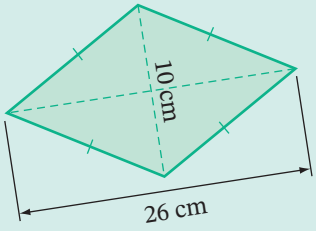
Example 7

Find the area of each shape.

a



b



Solution

a $A = \frac{1}{2}xy$

$= \frac{1}{2} \times 18 \times 52$

$= 468 \text{ cm}^2$

b $A = \frac{1}{2}xy$

$= \frac{1}{2} \times 10 \times 26$

$= 130 \text{ cm}^2$

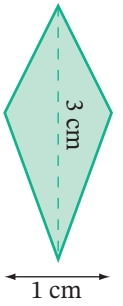
EXERCISE 5.06 ANSWERS ON P. 557

Areas of kites and rhombuses U F

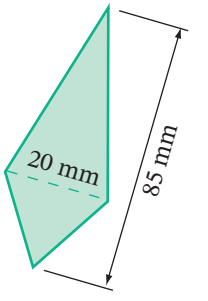
EXAMPLE 7

1 Find the area of each kite.

a



b



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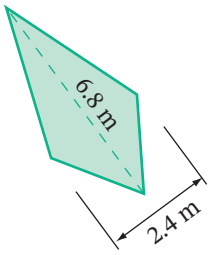
c



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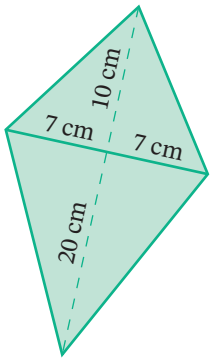
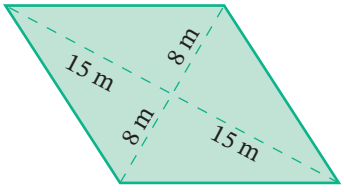


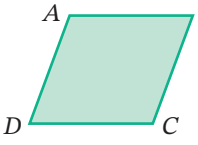
2 Find the area of each rhombus.

EXAMPLE 7

a

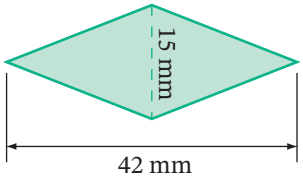


b

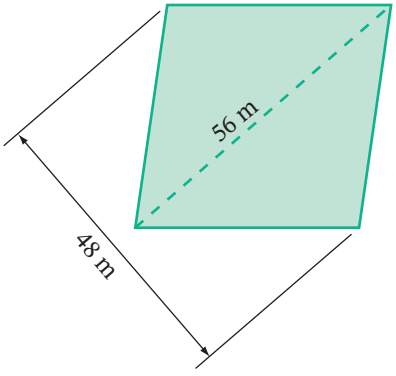


$AC = 10$ cm
$BD = 16$ cm

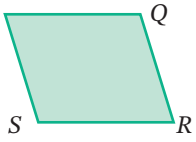
c



d



e



$PR = 2.4$ m
$QS = 1.5$ m

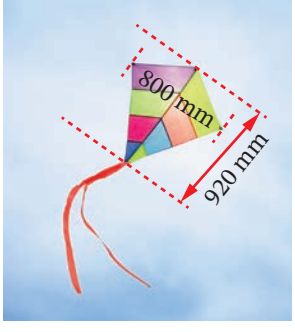
3 What area of material is needed to build this kite?
Select **A**, **B**, **C** or **D**.

A 36.8 cm^2

B 3680 cm^2

C 73.6 cm^2

D 7360 cm^2



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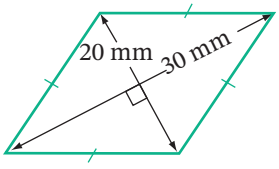
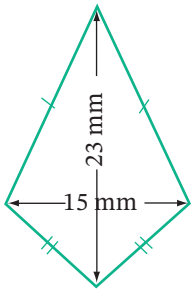
4 Find the area of each shape.

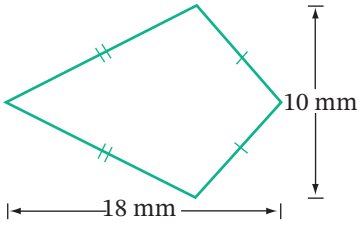
a



b



c



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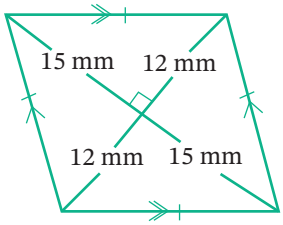
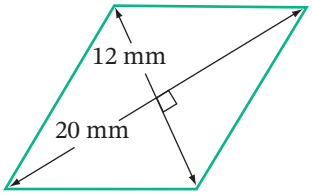
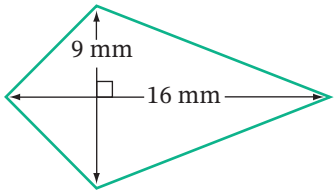


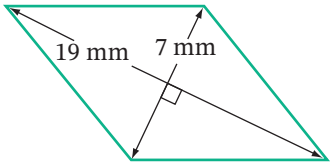
e



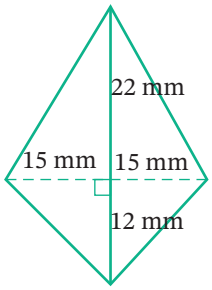
f



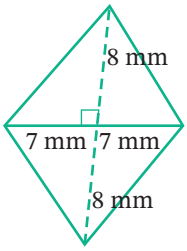
g



h



i

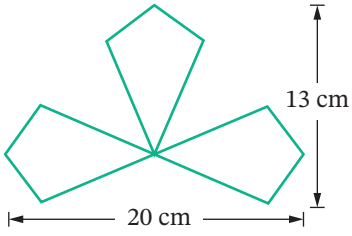


5 Amanda used 3 identical kites to draw this logo.

a What are the lengths of the diagonals for each kite?

b Find the area of one kite.

c Find the area of the entire logo.



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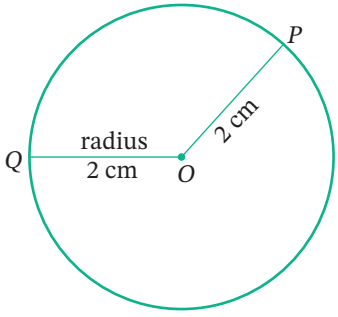
Parts of a circle

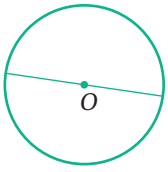
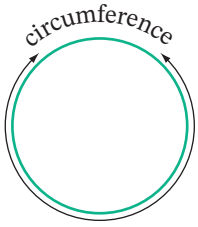
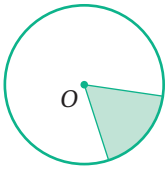
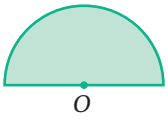
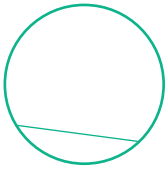
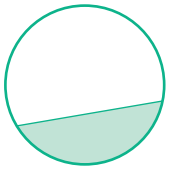
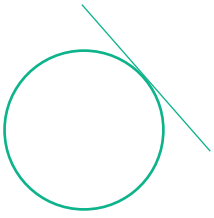
5.07

The circle is a completely round shape. Every point on a circle is the same distance from its centre. The circle shown has centre O, and points P and Q are exactly 2 cm from O. This distance, from the centre to the edge of the circle, is called the **radius** of the circle (*plural*: radii).



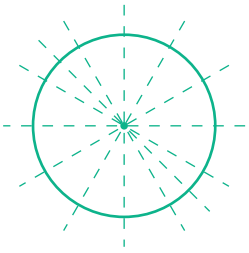
Circle words

There are special words that describe the parts of a circle or the related lines.

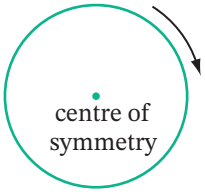
Diameter	Circumference	Arc
An interval running from one side of a circle to the other side and through the circle's centre 	The perimeter of a circle 	Part of the circumference 
Sector	**Quadrant**	**Semicircle**
A 'pizza-slice' of a circle cut along 2 radii 	A sector that is a quarter of a circle, with angle 90° 	Half a circle 
Chord	**Segment**	**Tangent**
An interval joining any 2 points on a circle 	A region cut off by a chord 	A line outside a circle that touches it at exactly one point 

Circle symmetry

A circle can be folded in half in an infinite number of ways, along any diameter, so it has an *infinite* number of axes of symmetry.



A circle can also be rotated through any angle size and still map onto itself, so it has an *infinite* order of rotational symmetry. The centre of symmetry is the centre of the circle.



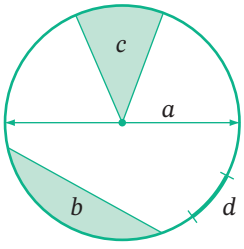
EXERCISE 5.07 ANSWERS ON P. 557

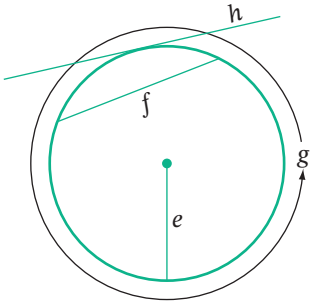
Parts of a circle **U F R C**

1 Use a ruler and compasses to construct a circle of:

a radius 4 cm **b** radius 2.5 cm **c** radius 56 mm

2 Name each part of a circle marked by letters in these diagrams. **C**





3 Match each description to the correct word from this list. **C**

circumference	radius	sector
tangent	arc	quadrant
segment	diameter	chord

a The distance from the centre of a circle to its side

b A quarter of a circle

c A line that touches the outside of a circle once

d The distance from one side of a circle to the other side, through the circle's centre

e A line segment from one side of the circle to the other side, not through the centre

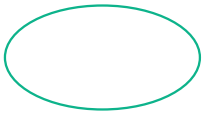
f Part of the circumference of a circle

g The area inside a circle formed by 2 radii and an arc

h The distance around a circle

i The area inside a circle formed by a chord and an arc

4 This ellipse (oval) is a 'flattened' circle. How many axes of symmetry has an ellipse? Select the correct answer **A**, **B**, **C** or **D**. R C



A 1 **B** 2

C 4 **D** an infinite number

5 What is the order of rotational symmetry of an ellipse? Select **A**, **B**, **C** or **D**. R C

A 1 **B** 2 **C** 4 **D** infinite number

6 Sketch a diagram of each term. C

a diameter **b** circumference **c** chord

d sector **e** tangent **f** segment

g quadrant **h** arc **i** radius

7 Copy each shape. Then state: R C

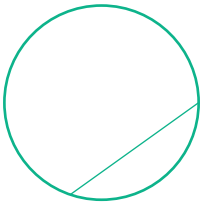
i the number of axes of symmetry in the shape and draw them

ii the order of rotational symmetry of the shape if it has rotational symmetry.

a

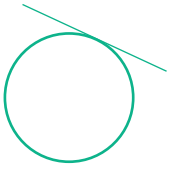
b

c



d

e



f

8 Using a radius of 4 cm, draw:

a a semicircle

b a quadrant

c a sector whose angle at the centre of the circle is 60°

d an arc that is longer than half the circumference of the circle

e a circle with a chord of length 2.5 cm

f a circle with a tangent of length 4 cm.

5.07

Technology

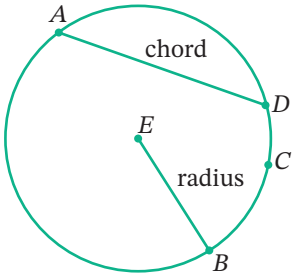
Circle parts

In this activity you will use your dynamic geometry software to construct a circle and display all the circle parts.

1 Construct a large circle, radius 4 cm.

2 Draw a radius and a chord in your circle.

3 Label the chord and the radius as shown.



4 Construct each circle part below on your page and label each one.

centre	diameter	circumference	sector	arc
segment	semicircle	tangent	quadrant	

5 What are **concentric circles**? Find out and construct a diagram showing 3 or more concentric circles. Remember to clearly label the centre of the circle.

Investigation

Measuring the circumference of a circle

WS Discovering pi

This is a measuring activity to be done in groups of 3.

You will need: A measuring tape, or some string and a ruler; graph paper; 6 round objects such as cans, round cake tins, pipes, coins, drums, bottles, and so on.

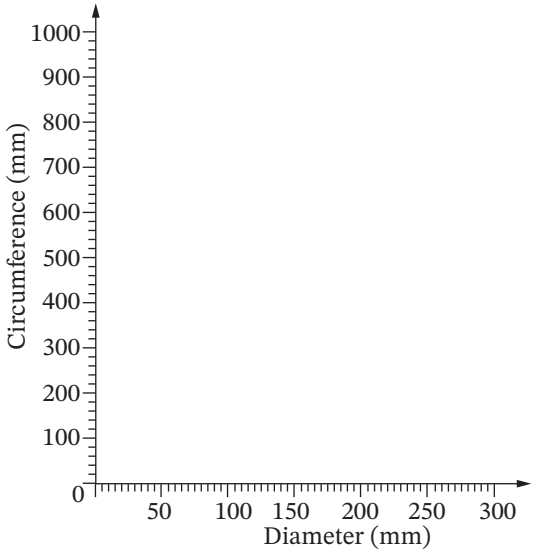
1 Copy this table.

Object	Diameter, d (mm)	Circumference, C (mm)	The ratio $\frac{C}{d}$
a			
b			
c			
d			
e			
f			

2 Measure (in millimetres) to find the diameter and the circumference of 6 different objects. Record your results in the table.

3 Calculate the ratio $\frac{C}{d}$ for each object. Round your answers to 2 decimal places.

4 Copy and complete the graph, using your results from the table. What do you notice about the graph?



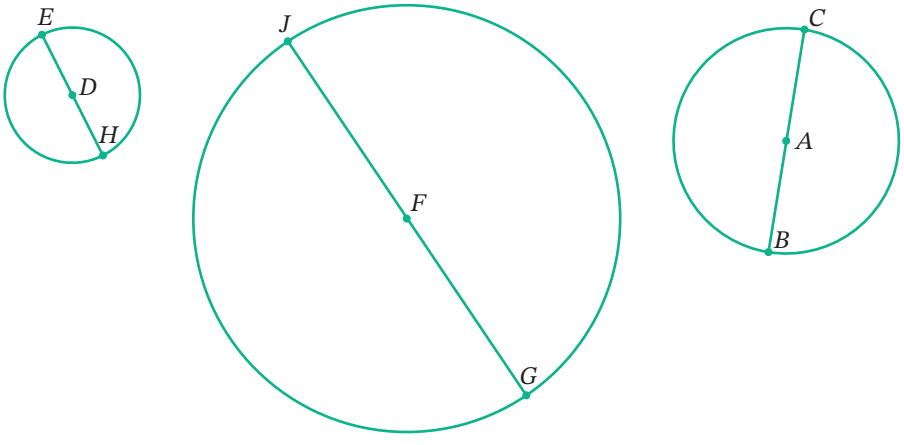
5 There is a formula for finding the circumference of a circle.

Copy and complete the formula: $C = d \times$ _____

Technology

The circumference and diameter of a circle

1 Use your dynamic geometry software to construct 3 different circles. Construct the diameter of each circle.



2 Calculate each circle's circumference and diameter.

3 For each circle, use your calculator to find $\frac{\text{length of circumference}}{\text{length of diameter}}$

4 What do you notice? Discuss this with another student.

5.08 Circumference of a circle

WS A page of circles

WS Circumference and area

Circle crossword

The **circumference** of a circle can be found by multiplying its **diameter** by a special number called **pi** (pronounced 'pie'), represented by the Greek letter π. For any circle, $\frac{\text{circumference}}{\text{diameter}} = \pi = 3.14\ldots$

Pi is often estimated as 3.14, but a more accurate value can be found on your calculator when you press the π key (you may need to press SHIFT first). As a decimal, the digits of π continue endlessly without any repeats or patterns:

$\pi = 3.141\ 592\ 653\ 589\ 793\ ...$

so, like $\sqrt{2}$, it is called an **irrational number**.

This number was named 'pi' by the Swiss mathematician Leonhard Euler in 1737.

The formula for the circumference of a circle is $C = \pi \times \text{diameter} = \pi d$.

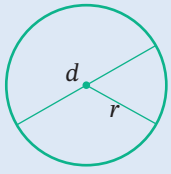
Because the diameter of a circle is double its radius, the circumference, C, of a circle with radius r is $C = \pi \times 2 \times \text{radius} = 2\pi r$.

Circumference of a circle

The **circumference (perimeter) of a circle** is:

$C = \pi \times \text{diameter}$ or $C = 2 \times \pi \times \text{radius}$

$C = \pi d$ or $C = 2\pi r$



Circumference of a circle

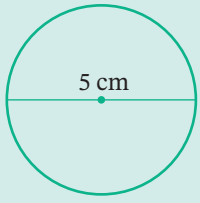
Example 8

a Estimate the circumference of a circle with a diameter of 5 cm.

b Calculate the circumference of the circle:

i correct to 2 decimal places

ii in terms of π



Solution

a $C = \pi d$

$= \pi \times 5$

$\approx 3 \times 5$ (π is approximately 3.)

$= 15$ cm

b **i** $C = \pi d$ On your calculator enter π × 5 =.

$= \pi \times 5$

$= 15.707\ 963\ldots$

≈ 15.71cm

ii $C = \pi d$

$= \pi \times 5$

$= 5\pi$ cm

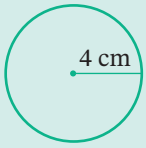
Like writing an answer as a surd in Pythagoras' theorem problems, writing an answer in terms of π is more exact as there is no rounding involved.

Example 9

Calculate the circumference of the circle:

a correct to one decimal place

b in terms of π



Solution

a $C = 2\pi r$
$= 2 \times \pi \times 4$
$= 25.13274\ldots$
≈ 25.1 cm

b $C = 2\pi r$
$= 2 \times \pi \times 4$
$= 8\pi$ cm

Circumference of a circle

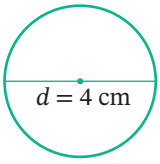
5.08

EXERCISE 5.08 ANSWERS ON P. 558

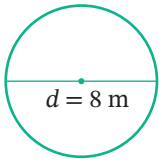
Circumference of a circle U F PS R

1 Estimate the circumference of each circle.

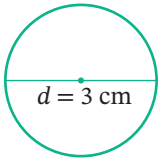
a



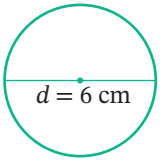
b



c



d



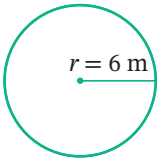
EXAMPLE 8

2 Calculate, correct to 2 decimal places, the circumference of each circle in question **1**.

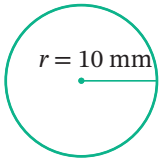
3 Calculate the circumference of each circle in question **1** in terms of π.

4 Estimate the circumference of each circle.

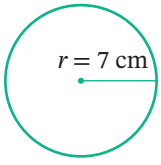
a



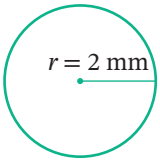
b



c



d



EXAMPLE 9

5 Calculate, correct to one decimal place, the circumference of each circle in question **4**.

6 Calculate the circumference of each circle in question **4** in terms of π.

7 A child's inflatable swimming pool has a diameter of 1.4 m. Find its circumference correct to 2 decimal places.

8 Tina's bicycle has wheels with a diameter of 60 cm. PS

a How far does the bicycle move when a wheel turns around once?

b If Tina cycles 900 m to school, how many complete turns does the bicycle wheel make?

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9 The Earth has a radius of 6370 km. Find the distance around the Equator.

10 A 20-cent coin has a radius of 16 mm. Calculate its circumference.

11 This tin of tomatoes has a diameter of 75 mm. If the label wraps around the tin completely, how long is the label? Answer correct to the nearest millimetre. PS R

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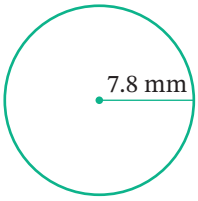
12 Calculate, correct to 2 decimal places, the circumference of a circle with:

a a diameter of 2 cm **b** a diameter of 7 cm

c a radius of 3 cm **d** a radius of 5 cm

13 Which of the following intervals best shows the circumference of this circle? Select **A**, **B**, **C** or **D**. R



A _____

B _____

C _____

D _____

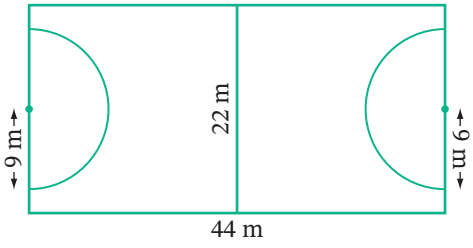
14 The International Space Station completes one circular orbit of the Earth at a height of 400 km. If the radius of the Earth is approximately 6400 km, find the distance travelled by the station. Select the closest answer **A**, **B**, **C** or **D**.

A 2513 km **B** 37 699 km **C** 40 212 km **D** 42 726 km

15 Tape has been placed on all the lines of this indoor hockey pitch. How much tape was used? Express the answer: PS R

a correct to one decimal place

b in terms of π.

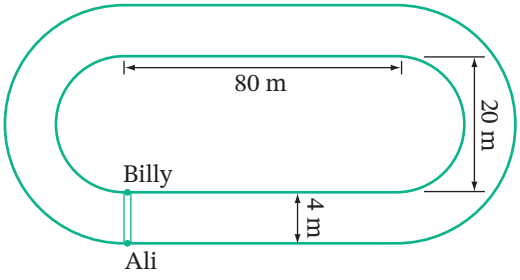


16 A circle has a circumference of 50.265 cm. Find its diameter, correct to the nearest centimetre. R

Foundation Standard Complex

9780170453141

17 Ali and Billy raced each other around this athletic track. Ali ran along the outside perimeter while Billy ran along the inside perimeter. After one lap of the track, who ran the longer distance, and by how much? Answer correct to the nearest metre. **PS** **R**



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5.08

Mental skills 5B: Maths without calculators ANSWERS ON P. 558

Comparing fractions, decimals and percentages

To compare or order fractions, express them with a common denominator first. To compare or order decimals, express them with the same number of decimal places first.

1 Study each example.

a Which fraction is smaller: $\frac{4}{10}$ or $\frac{3}{8}$?

Using a common denominator of 80 (8×10)

$\frac{4}{10} = \frac{4 \times 8}{10 \times 8} = \frac{32}{80}$

$\frac{3}{8} = \frac{3 \times 10}{8 \times 10} = \frac{30}{80}$

By comparing numerators: $\frac{30}{80} < \frac{32}{80}$

$\therefore \frac{3}{8}$ is smaller.

Using a common denominator of 40 (the LCM of 10 and 8)

$\frac{4}{10} = \frac{4 \times 4}{10 \times 4} = \frac{16}{40}$

$\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$

By comparing numerators: $\frac{15}{40} < \frac{16}{40}$

$\therefore \frac{3}{8}$ is smaller.

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b Arrange these decimals in ascending order: 0.407, 0.47, 0.047, 0.4.

Express all decimals with 3 decimal places by inserting zeros at the end where necessary.

0.407 0.470 0.047 0.400

By comparing digits in the same decimal place, the decimals from smallest to largest are: 0.047 0.400 0.407 0.470

That is, 0.047, 0.4, 0.407, 0.47

2 Now find the smaller number in each pair.

a $\frac{2}{3}$ and $\frac{3}{5}$ **b** $\frac{1}{4}$ and $\frac{1}{3}$ **c** 0.15 and 0.105

d 3.826 and 3.68 **e** $\frac{3}{7}$ and $\frac{4}{10}$ **f** $\frac{5}{6}$ and $\frac{7}{8}$

g 2.87 and 2.817 **h** 0.5301 and 0.503 **i** $\frac{1}{5}$ and $\frac{2}{9}$

3 Arrange each set of numbers in ascending order.

a 0.81, 0.082, 0.821 **b** 3.5, 3.51, 3.55, 3.513 **c** $\frac{2}{3}, \frac{1}{6}, \frac{2}{5}$

d 0.007, 0.07, 0.7, 0.707 **e** $\frac{3}{8}, \frac{2}{10}, \frac{1}{2}$ **f** $\frac{1}{4}, \frac{4}{10}, \frac{3}{5}$

4 Study this example.

Arrange these numbers in descending order: 68%, $\frac{13}{20}$, $0.\dot{6}$, $\frac{3}{5}$.

To order fractions, decimals and percentages, express them all as percentages first.

As percentages: $68\% = 68\%$, $\frac{13}{20} = 65\%$, $0.\dot{6} = 66.66...\%$, $\frac{3}{5} = 60\%$.

Largest to smallest, this is 68%, 66.66%, 65%, 60%, that is, 68%, $0.\dot{6}$, $\frac{13}{20}$, $\frac{3}{5}$

5 Now arrange each set of numbers in descending order.

a 0.25, $\frac{1}{6}$, 16%, $\frac{1}{5}$ **b** 27%, $\frac{1}{3}$, 0.4, 0.28 **c** 0.05, 50%, 6%, $\frac{1}{8}$

d $\frac{3}{4}$, 0.639, 55%, $\frac{2}{5}$ **e** 69%, 0.609, $\frac{2}{3}$, 0.6 **f** $\frac{2}{9}$, 0.105, 17%, 22.5%

Did you know?

Never-ending pi!

$\pi = 3.141\ 592\ 653\ 589\ 793\ 238\ 462\ 643\ 383\ 279\ 502\ 884\ 197\ 169\ 399\ 375\ 105\ 820...$

Ancient civilisations knew about the value of π, estimating it as 3. Over time, the calculations have improved due to better formulas and technology. Since the first computer, the ENIAC, was invented in 1949, much progress has been made. In 2019 Emma Haruka Iwao, a Google employee from Japan, calculated the value of π to 31.4 trillion digits with the help of Google's cloud computing service.

Year	Person/Country	Number of decimal places
1855	Shanks, England	527
1949	ENIAC computer, USA	2037
1973	CDC 7600 computer, France	1 000 000
1988	Kanada, Hitachi S-820 computer, Japan	200 000 000
1989	Chudnovsky brothers, USA	1 000 000 000
1999	Kanada and Takahashi, Hitachi SR8000 computer, Japan	206 158 430 000
2002	Kanada, Hitachi SR8000 computer, Japan	1 241 100 000 000
2010	Shigeru Kondo, Alexander Yee, Japan and USA	5 trillion
2019	Emma Haruka Iwao	31.4 trillion
???	???	???

The Guinness World Record for reciting the most digits of pi belongs to Rajveer Meena of India, who recited pi to 70 000 decimal places (while blindfolded) in 2015. Unofficially, in 2006, Japanese counsellor Akira Haraguchi memorised π to 100 000 decimal places. It took him over 16 hours to recite it. He had to stop after 3 hours as he lost his place, and had to start from the beginning.

On average, how many digits would Haraguchi have recited per minute?

Investigation

Belt around the Equator

Imagine that we wrapped a giant belt tightly around the Earth, along the Equator. This belt would touch the Earth at all points on its circumference, assuming the Earth was a perfect sphere or ball (with no mountains or valleys).

Now suppose we added one extra metre to the length of the belt. Then it would become loose and not touch the Earth any more. There would be a small gap between the Earth and the belt. How wide is this gap?

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1 In a group of 2 to 4 people, guess whether you could:

- **a** slip your hand between the belt and the ground
- **b** crawl under the belt
- **c** sit under the belt
- **d** stand under the belt.

2 If the diameter of the Earth is 12 755 metres, calculate to 2 decimal places, the length of the tight belt.

3 Add 1 metre to the length of the tight belt.

4 Calculate the diameter of the longer, looser belt.

5 So what is the length of the gap between the Earth and the loose belt?

6 Check whether your guesses in question **1** were correct.

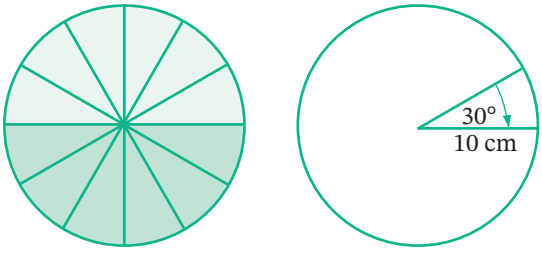
Investigation

WS
Approximating the area of a circle

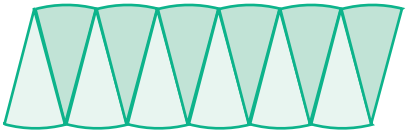
Area by cutting out sectors

You will need: compasses, a ruler, a protractor and a pair of scissors.

1 Draw a circle of radius 10 cm. Use your protractor to divide the circle into 12 sectors, each with angle size 30°.



2 Cut out all the sectors and arrange them into a shape like a parallelogram.

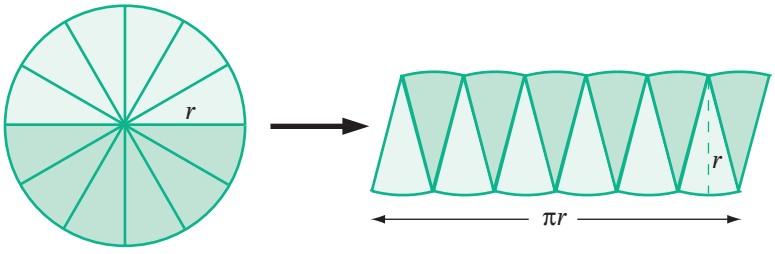


3 Use a ruler to measure the *base* and the *height* of your 'parallelogram'.

4 Now calculate the area of the 'parallelogram'.

5 Which formula gives the better approximation for the area of this circle: $A = 3 \times r^2$, or $A = 4 \times r^2$?

6 We will now find the actual formula for the area of a circle. A circle of radius r is cut up into many sectors and rearranged into a 'parallelogram'.



What is the formula for the circumference of this circle?

7 Explain why the length of the 'parallelogram' is πr.

8 What is the formula for the area of this 'parallelogram'?

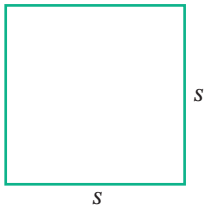
9 Explain why the area of the circle is πr^2.

Area of a circle

5.09

The area of a square is calculated by squaring its length.
$A = s \times s = s^2$

In a similar way, the area of a circle is calculated by squaring its radius and multiplying by π.



WS Circumference and area

WS Area ID

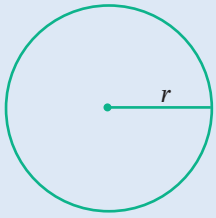
WS Area and perimeter investigations

Circle areas

Area of a circle

$A = \pi \times (\text{radius})^2$

$A = \pi r^2$

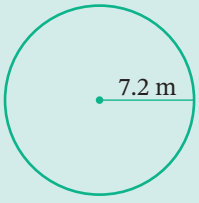


Example 10

Calculate the area of this circle:

a correct to 2 decimal places

b in terms of π



Solution

a $A = \pi r^2$

$= \pi \times 7.2^2$

$= 162.8601\ldots$

$\approx 162.86\ \text{m}^2$

Area is measured in square units, such as m^2

On a calculator enter: π × 7.2 x^2 =.

b $A = \pi r^2$

$= \pi \times 7.2^2$

$= 51.84\pi\ \text{m}^2$

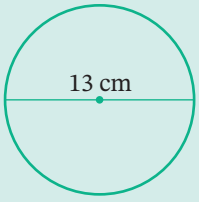
Area of a circle

Example 11

Find the area of this circle:

a correct to one decimal place

b in terms of π



Solution

a $r = \frac{1}{2} \times 13 = 6.5 \text{ cm}$

$A = \pi r^2$

$= \pi \times 6.5^2$

$= 132.7322\ldots$

$\approx 132.7 \text{cm}^2$

The radius is half of the diameter

b From **a**,

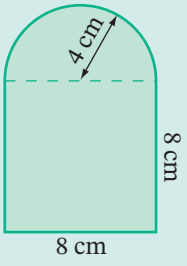
$A = \pi \times 6.5^2$

$= 42.25\pi \text{ cm}^2$

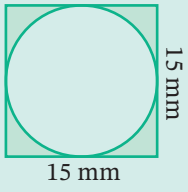
Example 12

Find, correct to 2 decimal places, the area of each shaded region.

a



b



Solution

a Area = area of square + area of semicircle

$= s^2 + \frac{1}{2} \times \pi r^2$

$= 8^2 + \frac{1}{2} \times \pi \times 4^2$

$= 89.132741\ldots$

$\approx 89.13 \text{ cm}^2$

b Shaded area = area of square − area of circle

$= s^2 - \pi r^2$

$= 15^2 - \pi \times 7.5^2$

$= 48.285413\ldots$

$\approx 48.29 \text{ mm}^2$

Radius: $r = \frac{1}{2} 15 = 7.5\text{mm}$

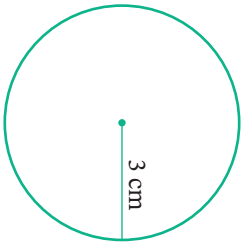
EXERCISE 5.09 ANSWERS ON P. 558

Area of a circle U F PS R

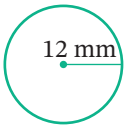
EXAMPLE 10

1 Calculate, correct to one decimal place, the area of each circle.

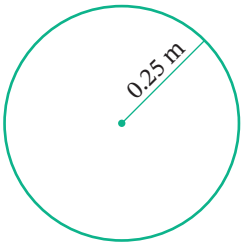
a



b



c



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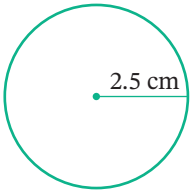


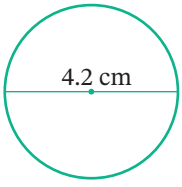
e

2 Calculate the area of each circle in question **1** in terms of π.

EXAMPLE 11

3 Calculate, correct to 2 decimal places, the area of each circle.

a



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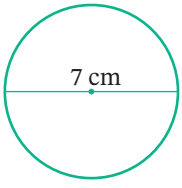

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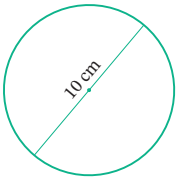


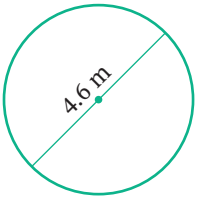
4 Calculate the area of each circle in question **3** in terms of π.

5 A sprinkler on the school playing field sprays water in a circular pattern of radius 13.1 m. Calculate the area being sprayed, to the nearest square metre.

6 Kevlar is a very strong light plastic. What area of it is needed to make a solid bicycle wheel of diameter 685.5 mm? Give your answer in square centimetres, correct to 2 decimal places.

7 A dinner plate has a radius of 14 cm. Calculate its area in terms of π.

8 What is the area of this circle, correct to one decimal place? Select the correct answer **A**, **B**, **C** or **D**.



A 14.5 m^2 **B** 66.5 m^2

C 7.2 m^2 **D** 16.6 m^2

Foundation Standard Complex

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9 The circular floor of a fishpond is to be covered in plastic. Find, correct to one decimal place, the area of plastic needed if the diameter of the pond is 2.8 m.

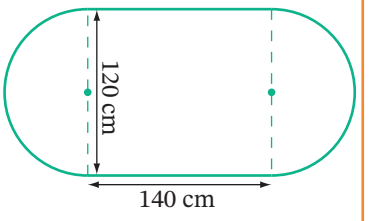
10 Find the approximate radius of a circular tree trunk whose cross-sectional area is 454.40 cm^2. Give your answer correct to 2 decimal places. R

EXAMPLE 12

11 The area of a DVD is 113 cm^2. Which of the following is closest to its diameter? Select **A**, **B**, **C** or **D**. R

A 12 cm **B** 13 cm **C** 12.5 cm **D** 13.1 cm

12 What is the area of this shape closest to? Select **A**, **B**, **C** or **D**. PS R

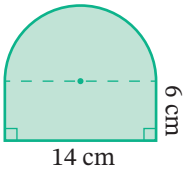


A 28 110 cm^2 **B** 22 455 cm^2

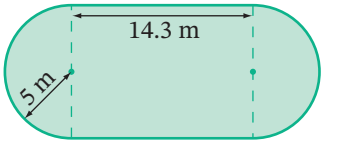
C 62 039 cm^2 **D** 11 570 cm^2

13 Find the area of each shaded region, correct to 2 decimal places. PS R

a

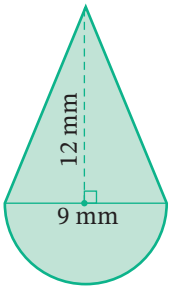


b



Hint: Combine 2 semicircles to make a whole circle.

c



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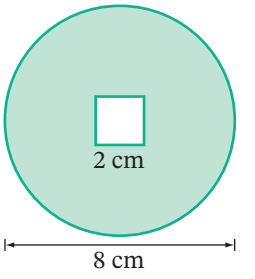

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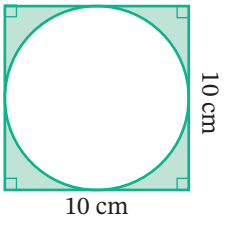

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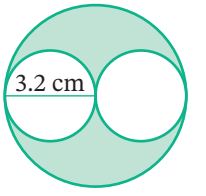
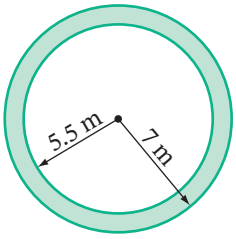
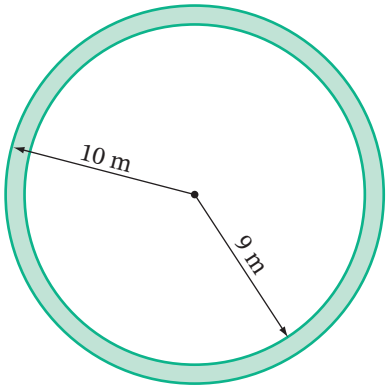


14 Each ring-shape or donut-shape shown below is called an **annulus**, made from 2 circles that are concentric (with the same centre) but of different size.

a



b



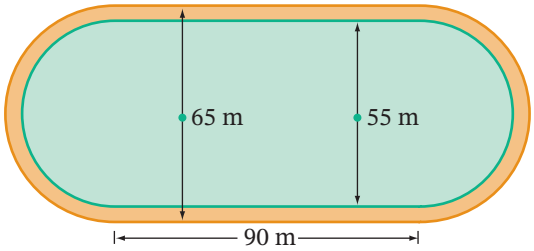
For each annulus, find, correct to one decimal place, the area of: R

i the larger circle

ii the smaller circle

iii the (shaded) annulus

15 The dimensions of a running track are shown. The ends are circular. PS R



a Find the combined area of the (orange) track and the (green) grassed central area.

b Find the area of the grassed area.

c Find the area of the track.

d It is decided to cover the track with synthetic grass that costs $24.25 per square metre. How much will the track surface cost?

5.10 Perimeter and area of a sector

Perimeter and area of a sector

A page of circular shapes

Applications of area

Back-to-front problems

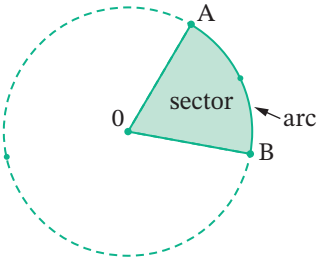
Area and perimeter investigations

Carpet talk

A sector is a fraction of a circle.

An arc is a fraction of a circle's circumference.

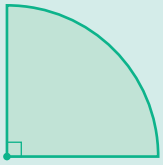
As there are 360° in a circle, the fraction is $\frac{\text{angle at the centre}}{360°}$.



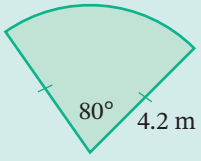
Example 13

Find, correct to 2 decimal places, the perimeter and area of each shape.

a



b



Solution

a This is a quadrant as the angle at the centre is 90°.

Fraction of circle is $\frac{90°}{360°} = \frac{1}{4}$

Perimeter = length of arc + radius + radius

Length of arc = $\frac{1}{4}$ × circumference of circle

$= \frac{1}{4} \times 2\pi r$

$= \frac{1}{4} \times 2 \times \pi \times 10$

$= 15.70796\ldots$

Perimeter of sector = 10 + 10 + 15.70796...

= 35.70796...

≈ 35.71 mm

Area of sector = $\frac{1}{4}$ × area of circle

$= \frac{1}{4} \times \pi \times 10^2$

$= 78.5398\ldots$

$\approx 78.54\ \text{m}^2$

b Angle at the centre is 80°.

Fraction of circle is $\frac{80°}{360°} = \frac{2}{9}$

Perimeter = length of arc + radius + radius

Length of arc = $\frac{2}{9}$ × circumference of circle

$= \frac{2}{9} \times 2\pi r$

$= \frac{2}{9} \times 2 \times \pi \times 4.2$

$= 5.8643\ldots$

Perimeter of sector = 4.2 + 4.2 + 5.8643...

= 14.2643...

≈ 14.26 m

Area of sector = $\frac{2}{9}$ × area of circle

$= \frac{2}{9} \times \pi \times 4.2^2$

$= 12.3150\ldots$

$\approx 12.32\ \text{m}^2$

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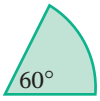
EXERCISE 5.10 ANSWERS ON P. 558

Perimeter and area of a sector U F PS R

1 What fraction of a whole circle is each sector?

a 

b 

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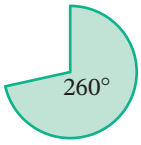
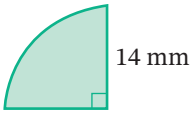
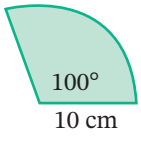
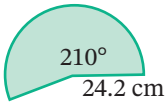
d 

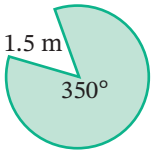
2 Find the perimeter of each sector, correct to one decimal place.

EXAMPLE 13

a 

b 

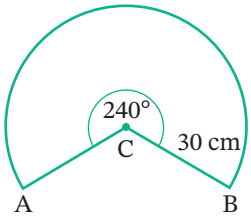
c 

d 

3 Find the area of this shape, correct to the nearest square centimetre. Select the correct answer **A**, **B**, **C** or **D**.

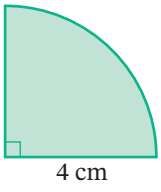
A 40 cm^2 **B** 63 cm^2

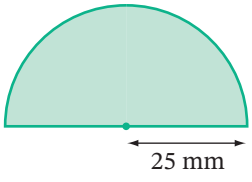
C 1200 cm^2 **D** 1885 cm^2



4 Find the area of each sector in question **2** correct to 2 decimal places.

5 Find, correct to one decimal place, the perimeter and area of each shape.

a 

b 

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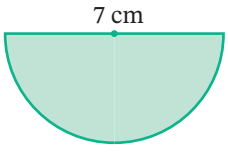
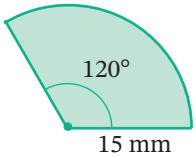
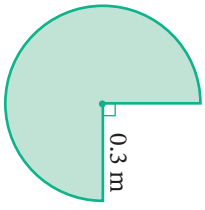
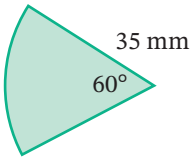
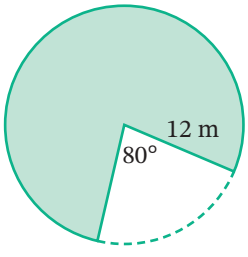
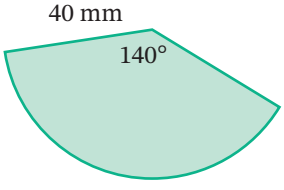
d 

e 

f 

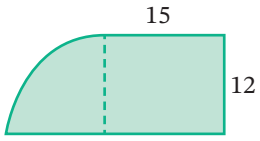
g 

h 

i 

Foundation Standard Complex

6 Find the perimeter of this shape, correct to the nearest whole square unit. Select the correct answer **A**, **B**, **C** or **D**.



A 64 **B** 63

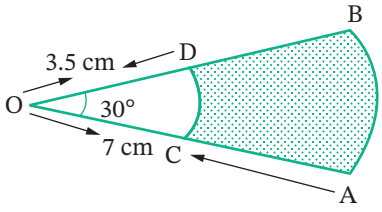
C 61 **D** 73

7 The length of the minute hand on a clock is 14 cm. PS R

a How many degrees does the minute hand rotate through in one minute?

b Find, correct to 2 decimal places, the area swept by the minute hand in one minute.

8 Find the area of the shaded section, corrrect to 2 decimal places. R



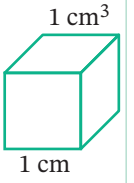
5.11 Metric units for volume

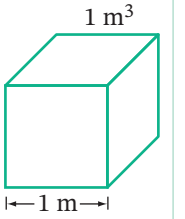
What is volume?

WS Volume

The **volume** of a solid is the amount of space it takes up. It is measured in **cubic units**.

cubic centimetre (cm^3)	cubic metre (m^3)
1 cm = 10 mm $1\,cm^3 = 1\,cm \times 1\,cm \times 1\,cm$ $= 10\,mm \times 10\,mm \times 10\,mm$ $= 1000\,mm^3$	1 m = 100 cm $1\,m^3 = 1\,m \times 1\,m \times 1\,m$ $= 100\,cm \times 100\,cm \times 100\,cm$ $= 1\,000\,000\,cm^3$





A **cubic centimetre** is about the volume of a person's tooth or a medical pill.

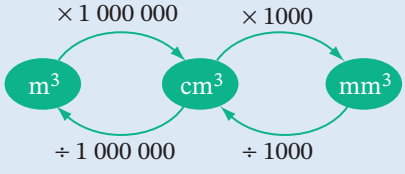
A **cubic metre** is about the volume of 2 washing machines.

Note that, while 1 cm = 10 mm, $1\,cm^3 = 1\,000\,mm^3$ (triple the number of 0s), and while 1 m = 100 cm, $1\,m^3 = 1\,000\,000\,cm^3$ (triple the number of 0s).

Metric units for volume

$1\,cm^3 = 1000\,mm^3$

$1\,m^3 = 1\,000\,000\,cm^3$



Foundation Standard Complex

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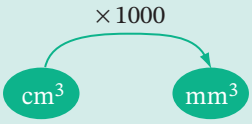
Example 14

Convert:

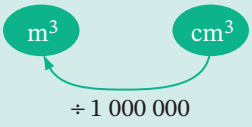
a 36 cm^3 to mm^3 **b** 84 000 000 cm^3 to m^3

Solution

a 36 $cm^3 = 36 \times 1000$ mm^3
$= 36\,000$ mm^3



b 84 000 000 $cm^3 = 84\,000\,000 \div 1\,000\,000$ m^3
$= 84$ m^3



EXERCISE 5.11 ANSWERS ON P. 559

Metric units for volume U F R C

1 What unit of volume should you use to measure the volume of: R C

a a bedroom? **b** a backpack?
c a mobile phone? **d** a concert hall?
e your body? **f** an aeroplane?

EXAMPLE 14

2 A swimming pool has a volume of 38 m^3. How many cubic centimetres is this pool? Select the correct answer **A**, **B**, **C** or **D**. R C

A 380 000 **B** 3 800 000
C 38 000 000 **D** 380 000 000

3 Copy and complete each conversion. R C

a 5000 cm^3 = _____ mm^3 **b** 1.6 m^3 = _____ cm^3
c 2 cm^3 = _____ mm^3 **d** 4 m^3 = _____ mm^3
e 6000 cm^3 = _____ m^3 **f** 8.2 m^3 = _____ cm^3
g 0.007 m^3 = _____ mm^3 **h** 9 600 000 mm^3 = _____ m^3
i 4000 mm^3 = _____ cm^3 **j** 160 000 cm^3 = _____ m^3
k 250 mm^3 = _____ cm^3 **l** 12 cm^3 =_____ mm^3
m 0.18 m^3 = _____ cm^3 **n** 200 000 cm^3 =_____ m^3

4 The volume of a chest of drawers is 272 000 cm^3. Convert this to cubic metres. R C

5 What is the approximate volume of a house brick? Select **A**, **B**, **C** or **D** R

A 1000 cm^3 **B** 20 cm^3 **C** 800 cm^3 **D** 2100 cm^3

6 How many cubic millimetres are there in 2.3 m^3? Select **A**, **B**, **C** or **D**. R C

A 2 300 000 000 **B** 2300 **C** 23 000 **D** 0.0023

7 Match the correct volume (**A** to **G**) with each item (**a** to **g**) listed. R

Shutterstock.com/Monkey Business Images

a	Mobile phone	**A**	200 m^3
b	Box of tissues	**B**	3890 m^3
c	Glass of water	**C**	1250 cm^3
d	Bottle of soft drink	**D**	5000 cm^3
e	Classroom	**E**	80 cm^3
f	School hall	**F**	250 cm^3
g	Box of cereal	**G**	2200 cm^3

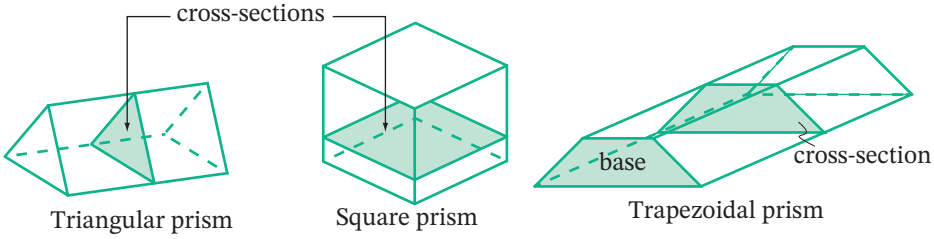
8 The volume of a lunchbox is 2460 cm^3. Convert this to cubic millimetres. R C

Volume of a prism

5.12

A **cross-section** of a solid is a 'slice' of the solid, cut *across* it, parallel to its end faces, rather than along it.

If a solid has the same (uniform) cross-section along its length, and each cross-section is a **polygon** (with straight sides, not rounded), then the solid is called a **prism**. Here are some examples of prisms:



Triangular prism | Square prism | Trapezoidal prism

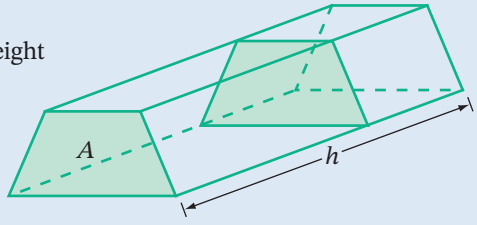
Prisms take their names from their cross-section. For example, the prism shown above is called a **trapezoidal prism** because its cross-sections are all trapeziums.

Because a prism is made up of identical cross-section 'slices' along its length, its volume can be calculated by multiplying the **area of the cross-section** by its **perpendicular height** (the length or depth of the prism).

Volume of a prism

Volume = area of cross-section × perpendicular height

$V = Ah$



Example 15

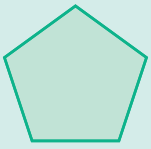
Convert:

a What is the name of this polygon?

b Draw a prism with this polygon as a cross-section.

c What shape are the side faces?

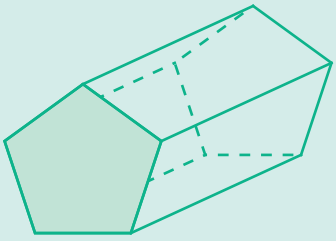
d What is the name of the prism?



Solution

a Pentagon

b



c The side faces are all rectangles.

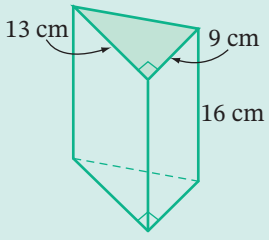
d This is a pentagonal prism.

Example 16

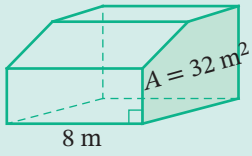
Volume of a prism

Find the volume of each prism.

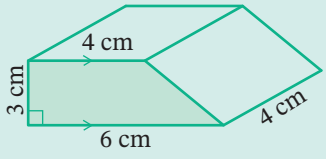
a



b



c



Solution

a $A = \frac{1}{2} \times 9 \times 13$ — Area of shaded triangular cross-section

$= 58.5\,\text{cm}^2$

$V = 58.5 \times 16$ — $V = Ah$, where height $h = 16$ cm

$= 963\,\text{cm}^3$

b $A = 32\,\text{m}^2$ — Area of shaded cross-section

$V = 32 \times 8$ — $V = Ah$, where height $h = 8$

$= 256\,\text{m}^3$

c $A = \frac{1}{2} \times (4+6) \times 3$ — Cross-section is a trapezium

$= 15\,\text{cm}^2$

$V = 15 \times 4$ — $V = Ah$, where height $h = 4$

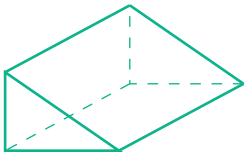
$= 60\,\text{cm}^3$

EXERCISE 5.12 ANSWERS ON P. 559

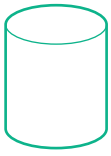
Volume of a prism U F PS R C

1 State whether each solid is a prism or not. C

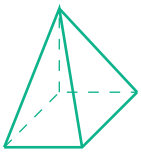
a



b



c



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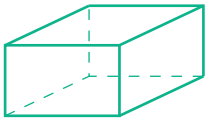
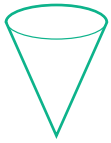


e



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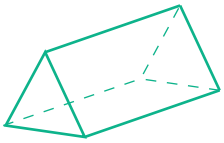


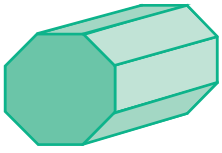
EXAMPLE 15

2 For each prism:

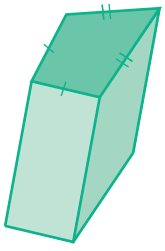
i draw its cross-section

ii write the name of the prism

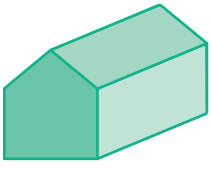
a



b



c



3 Draw each prism and shade its base.

a square prism

b isosceles triangular prism

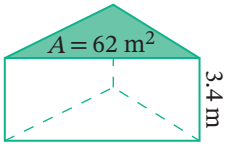
c trapezoidal prism

d hexagonal prism

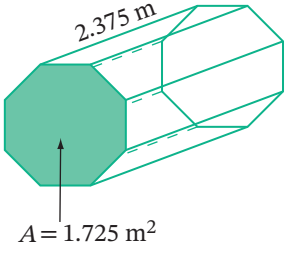
EXAMPLE 16

4 Find the volume of each prism.

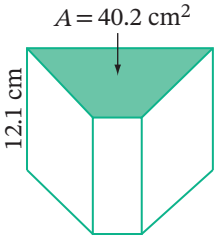
a



b



c



Foundation | Standard | Complex

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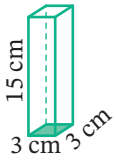
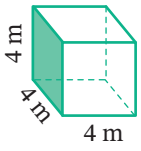
5 The volume of a rectangular prism can be found using the formula $V = lwh$.
Use this formula to find the volume of each prism below.

a



b



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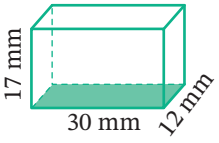

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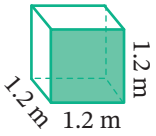
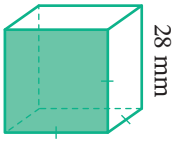
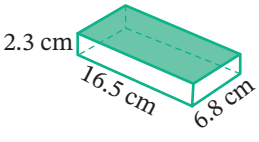


e

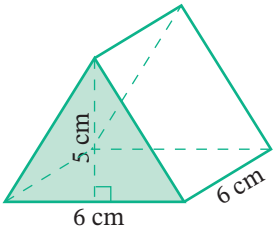


f

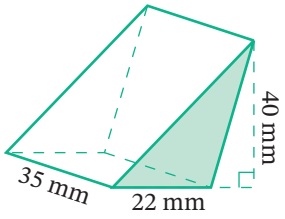


6 Find the volume of each triangular prism.

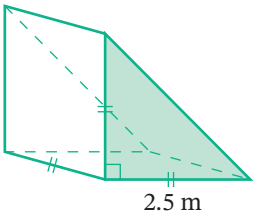
a



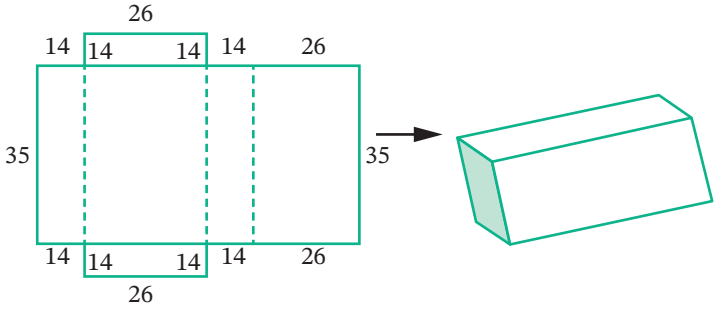
b



c



7 This shape is folded along the dotted lines to form a rectangular prism.
Find the volume of the box in cubic units. Select the correct answer **A**, **B**, **C** or **D**. R



A 23 660 **B** 12 740 **C** 5096 **D** 258

Foundation Standard Complex

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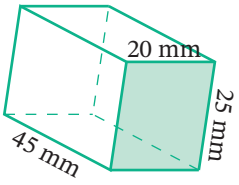
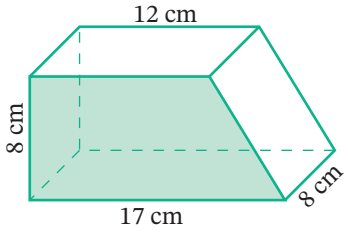
8 Find the volume of each prism.

a



b



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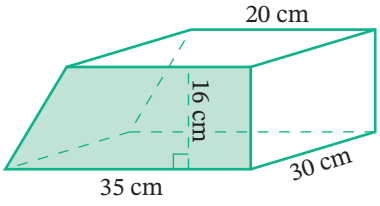
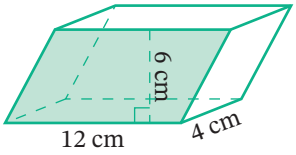
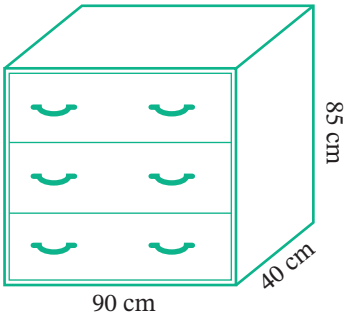


d



9 Find the volume of this chest of drawers. Give the answer in m^3: R C

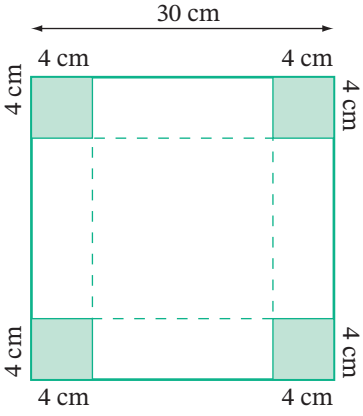


a by calculating the volume in cm^3, then converting to m^3

b by converting each length to metres first, then calculating the volume.

Which method is easier?

10 A square sheet of metal has a side length of 30 cm. 4 identical squares of length 4 cm are cut away from the corners, as shown. Find the volume of the container formed when the remaining shape is folded along the dotted lines. Select **A**, **B**, **C** or **D**. PS R



A 1936 cm^2

B 2704 cm^2

C 64 cm^2

D 3600 cm^2

11 A triangular prism has base length 6 m, height 10 m and volume 150 m^3. What is its length? R

12 A cube has a volume of 343 m^3. What is its side length? R

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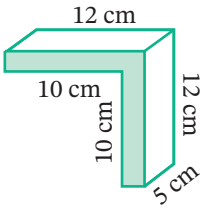
13 Calculate the volume of each prism PS R

a



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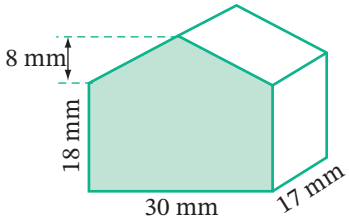

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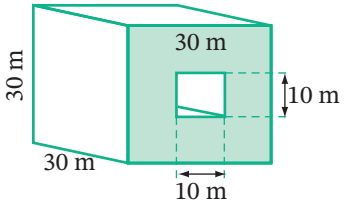

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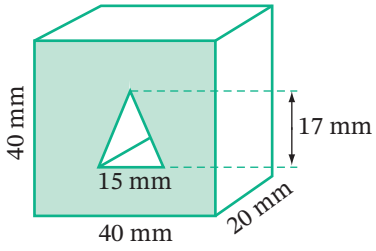

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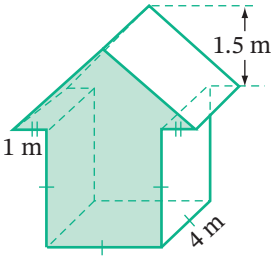


14 A rectangular garden is 12 metres long and 4 metres wide. It is filled with soil to a depth of 15 centimetres. Calculate the volume of the soil. Select **A**, **B**, **C** or **D**. PS R

A 0.72 m^3 **B** 7.2 m^3 **C** 72 m^3 **D** 720 m^3

15 A triangular prism has a volume of 36 cm^3. What could its length, perpendicular height and length be? R

5.13 Volume of a cylinder

Volumes of prisms and cylinders

A **cylinder** is like a 'circular prism' because its cross-sections are all identical circles. Because of this, we can also apply the prism formula $V = Ah$ to calculate the volume of a cylinder.

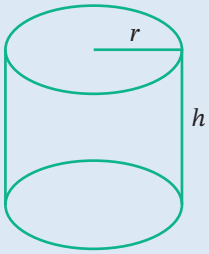
For a circle, $A = \pi r^2$, so:

Volume $= Ah = \pi r^2 \times h = \pi r^2 h$

Volume of a cylinder

$V = \pi \times (\text{radius})^2 \times \text{perpendicular height}$

$V = \pi r^2 h$



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Example 17

Find the volume of this cylinder, correct to one decimal place.

Solution

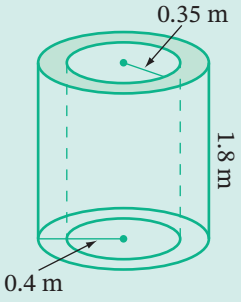
$V = \pi \times 5^2 \times 12.5$ $\qquad V = \pi r^2 h$

$= 981.7477\ldots$

$\approx 981.7 \text{ cm}^3$

Example 18

Calculate the volume of concrete needed to make this pipe, correct to 2 decimal places.



Solution

Volume of concrete = volume of big cylinder – volume of small cylinder (hole)

$= \pi \times 0.4^2 \times 1.8 - \pi \times 0.35^2 \times 1.8$

$= 0.212057\ldots$

$\approx 0.21 \text{ m}^3$

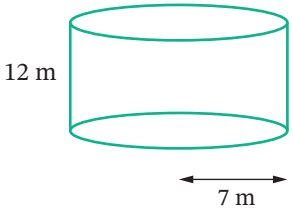
EXERCISE 5.13 ANSWERS ON P. 559

Volume of a cylinder **U F PS R**

1 A cylinder has a radius of 7 m and a height of 12 m. Find correct to 2 decimal places:

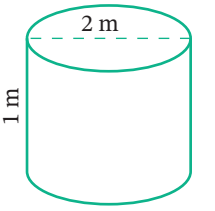
a the area of its circular base

b the volume of the cylinder.



EXAMPLE 17

2 What is the volume of this cylinder? Select the closest answer **A**, **B**, **C** or **D**.

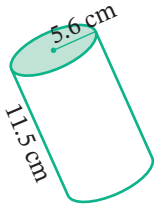


A $2\ m^3$ **B** $12\ m^3$

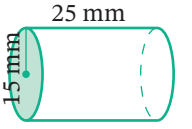
C $3\ m^3$ **D** $6\ m^3$

3 Find, correct to one decimal place, the volume of each cylinder.

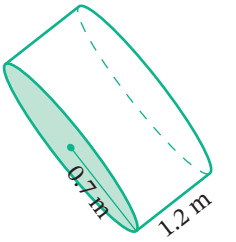
a



b



c



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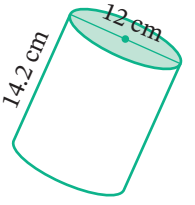

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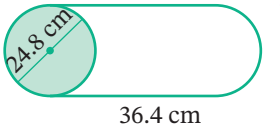

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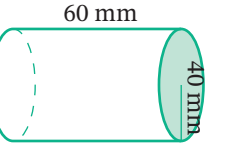

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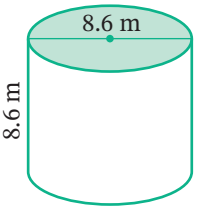
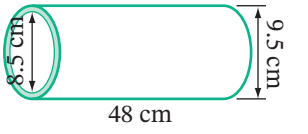


EXAMPLE 18

4 This metal pipe has an inner diameter of 8.5 cm and an outer diameter of 9.5 cm. PS R



a What is the inner radius?

b What is the outer radius?

c Calculate, correct to 2 decimal places, the volume of metal needed to make the pipe.

5 A manufacturer is experimenting with a new size of can. Which of these options provides the larger volume? PS R

A A can of radius 3.95 cm and height 11.8 cm

B A can of diameter 8.1 cm and height 15 cm

6 A cylinder has a radius of 5 cm and a volume of $628.32\ cm^3$. Find its height, correct to the nearest centimetre. R

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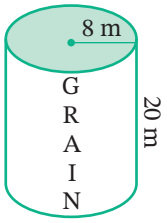
7 Find the volume of each solid, correct to one decimal place. PS R

a



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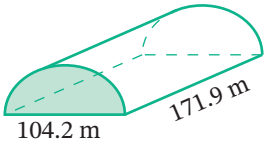

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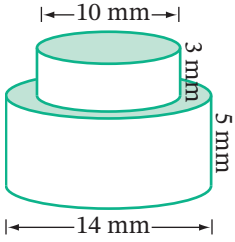

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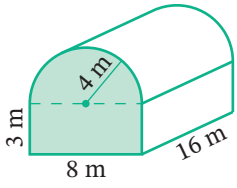

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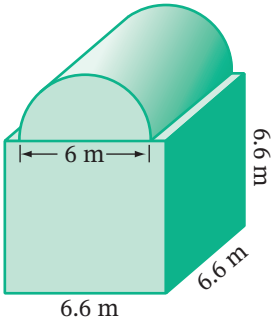

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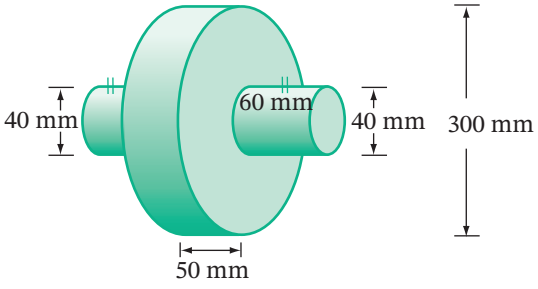

8 A cylinder has a volume of 552.92 m^3 and a height of 11 m. Find its radius, correct to the nearest metre. R

5.13

Volume and capacity 5.14

Capacity is the amount of fluid (liquid or gas) in a container, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML).

- A large drop of water is about 1 mL
- A teaspoon holds about 5 mL
- A tall standard carton of milk holds 1 L
- A small rainwater tank holds about 1 kL

A can of drink holds 375 mL.

Shutterstock.com/Oleksiy Mark

A can of paint holds 4 L.

Shutterstock.com/Hintau Aliaksei

WS Volume and capacity

WS Volume and capacity cards

WS Capacity

Capacity puzzle

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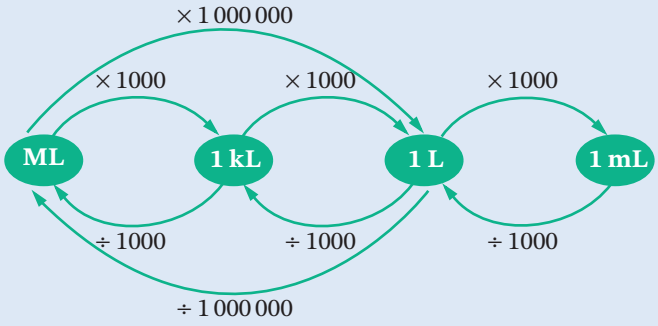
Volume and capacity

Metric units of capacity

1 L = 1000 mL

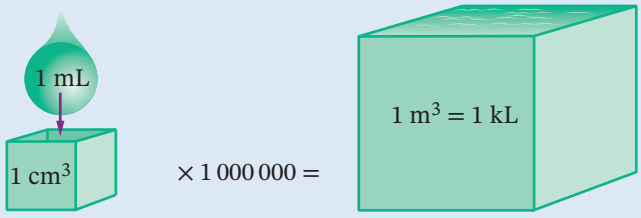
1 kL = 1000 L

1 ML = 1000 kL = 1 000 000 L



1 cm^3 contains 1 mL

1 m^3 contains 1000 L or 1 kL



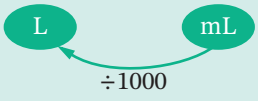
Example 19

Convert:

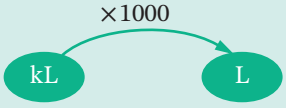
a 3400 mL to L **b** 2.9 m^3 to L

Solution

a 3400 mL = 3400 ÷ 1000 L
= 3.4 L

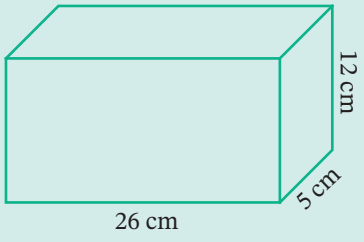


b 2.9 m^3 = 2.9 kL
= 2.9 × 1000 L
= 2900 L



Example 20

Find the capacity of this container in litres.



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Solution

$V = 32 \times 8 \times 15$
$= 3840\ \text{cm}^3$

$V = lwh$
$1\ \text{cm}^3 = 1\ \text{mL}$

Capacity $= 3840$ mL
$= 3840 \div 1000$ L
$= 3.84$ L
Capacity $= 3840$ mL or 3.84 L

EXERCISE 5.14 ANSWERS ON P. 559

Volume and capacity U F PS R C

1 State what unit of capacity you would use when measuring: R C

a a glass of water **b** a dam **c** a car's petrol tank
d a bottle of medicine **e** an office water cooler
f an Olympic swimming pool

2 Copy and complete each conversion. R C

EXAMPLE 19

a 5000 mL = _____ L **b** 3.4 kL = _____ L **c** 1.6 L = _____ mL
d 4000 cm^3 = _____ L **e** 2980 kL = _____ ML **f** 7.1 ML = _____ L
g 875 L = _____ kL **h** 8.2 m^3 = _____ L **i** 0.8 ML = _____ kL
j 1850 mL = _____ L **k** 5.4 kL = _____ L **l** 900 000 L = _____ m^3
m 6 kL = _____ L **n** 3500 mm^3 = _____ mL **o** 1.2 m^3 = _____ mL

3 What is the capacity of a regular bottle of cough medicine? Select the closest answer **A**, **B**, **C** or **D**. R

A 200 mL **B** 500 mL **C** 1500 mL **D** 2000 mL

4 The internal dimensions of a refrigerator are 150 cm (height), 60 cm (width) and 40 cm (depth). What is the capacity of the refrigerator? Select the closest answer **A**, **B**, **C** or **D**. R C

EXAMPLE 20

A 350 L **B** 400 L **C** 250 L **D** 300L

5 A rectangular fish tank is 90 cm long and 30 cm wide, and is filled to a depth of 45 cm. Find the volume of water in: PS R C

a cm^3 **b** litres

6 A can of drink has a radius of 3 cm and a height of 15 cm. Find its volume, correct to the nearest millilitre. PS R C

7 A swimming pool is the shape of a rectangular prism 50 m × 18 m × 2 m. Find its volume in litres. PS R C

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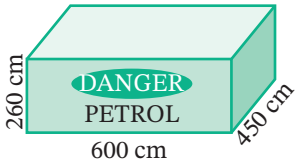
8 A tank in the shape of a rectangular prism has dimensions as shown. PS R C



a Find the volume of the tank in cm^3.

b How many litres of petrol would fit in the tank?

9 Samantha is inviting 18 friends to a party. She calculates that each person will drink 1300 mL of soft drink. PS R

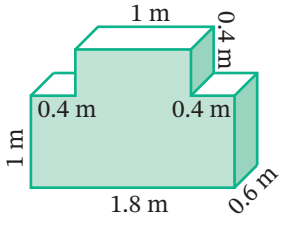
a How many litres of soft drink must she buy?

b If Samantha intends to buy large 2.5 L bottles of drink, how many bottles must she buy?

10 A gardener orders a cubic metre of topsoil. She wants to spread it in her garden to a depth of 10 cm. If her garden is rectangular, what might be its dimensions? PS R C

11 How many litres of rainwater were collected by a tank when 40 mm of rain fell on a roof 12 m long and 3.6 m wide? PS R C

12 A plastic road block barrier is shown. It is filled with water to weigh it down. PS R C



a Find the volume of the barrier.

b How many litres of water would it take to fill the barrier?

13 The tank on a fuel tanker is in the shape of a cylinder 6 metres long, with a diameter of 2 metres. PS R C

a Find, correct to 3 decimal places, the volume of the tank in cubic metres.

b How many litres of fuel can this tank hold?

Shutterstock.com/Krivosheev Vitaly

14 A tap leaks 10 mL of water every 50 seconds. How many litres of water will be lost in: PS R

a 1 hour? **b** 1 day?

15 A cylindrical can holds 2 L of pesticide. What could its radius and height be? PS R

Did you know?

Water is everywhere

Water is the most valuable resource in our planet. Here are some interesting facts about water.

- 68.7% of the fresh water on Earth is trapped in glaciers
- A jellyfish and a cucumber are made of 95% water
- 70% of the human brain is water
- A human can live about one month without food, but only about a week without water
- About 25 700 litres of water are needed to grow a day's food for a family of 4
- About 80% of a newborn baby's mass is water

How many megalitres of water are needed to feed a family of 4 for a year?

If a baby weighs 3.8 kg at birth, how much of this is water?

Shutterstock.com/Christina Fink

Wallaman Falls in Queensland, the tallest waterfall in Australia.

Investigation

Drinking the swimming pool dry

Charlotte's mum told her that it was healthy to drink 6 full glasses of water each day. Assume that Charlotte uses a cylindrical glass with a height of 9 cm and a diameter of 7 cm.

Shutterstock.com/Syda Productions

1 Calculate the volume of water Charlotte drinks each day, in millilitres, correct to 4 decimal places.

2 How much water will Charlotte drink in a year? Answer in litres, correct to 4 decimal places.

3 The family's swimming pool is in the shape of a cylinder with a diameter of 8.3 metres, and is filled to a constant depth of 1.2 metres. What is the amount of water in the pool? Answer in litres, correct to 4 decimal places.

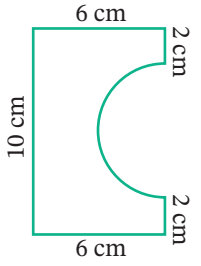
4 How long would it take Charlotte to drink the equivalent of the swimming pool? Select the closest answer **A**, **B**, **C** or **D**.

A 6 years **B** 26 years **C** 56 years **D** 86 years

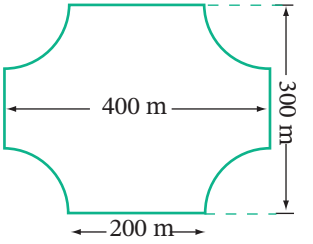
Power plus ANSWERS ON P. 559

1 Find, correct to one decimal place, the perimeter of each shape.

a



b



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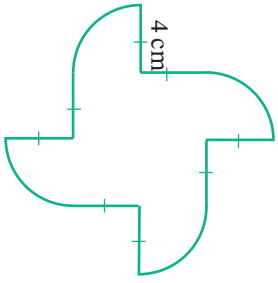
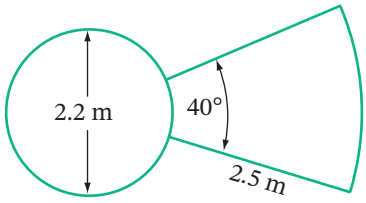


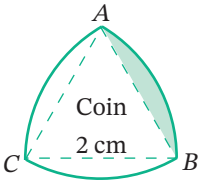
2 This diagram shows how a shotput field is marked out for an athletics event. Calculate, correct to one decimal place:

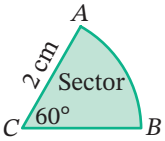


a the area of the field

b the total length of the lines used.

3 Mathsland has introduced a new 30-cent coin, as shown in the first diagram.
$\triangle ABC$ is an equilateral triangle of length 2 cm.





a Find, correct to 3 decimal places, the area of the sector in the second diagram.

b Use Pythagoras' theorem to prove that the perpendicular height of $\triangle ABC$ is $\sqrt{3}$ cm.

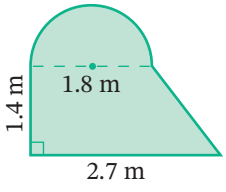
c Calculate the area of $\triangle ABC$, correct to 3 decimal places.

d Calculate the area of the shaded segment in the first diagram.

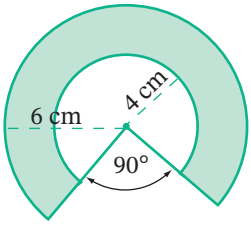
e Hence calculate the area of the 30-cent coin, correct to 2 decimal places.

4 Find, correct to one decimal place, the area of each shape.

a



b



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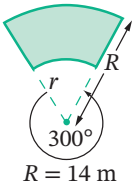

$R = 14$ m
$r = 11$ m

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CHAPTER 5 REVIEW

Language of maths

arc	base	capacity	chord
circumference	cross-section	cubic metre	cylinder
diameter	hectare	perimeter	perpendicular height
pi (π)	prism	quadrant	radius/radii
rectangular prism	sector	segment	semicircle
square metre	tangent	triangular prism	volume

Measuring shapes and time

1 Why are there 2 formulas for the circumference of a circle?

2 What is meant by the **perpendicular height** of a shape?

3 What is the correct name for a 'pizza slice' of a circle bounded by 2 radii and an arc?

4 In the formula $A = \frac{1}{2}xy$ for the area of a rhombus, what do x and y stand for?

5 What name is given to a 'slice' of a solid taken across the solid, rather than along it?

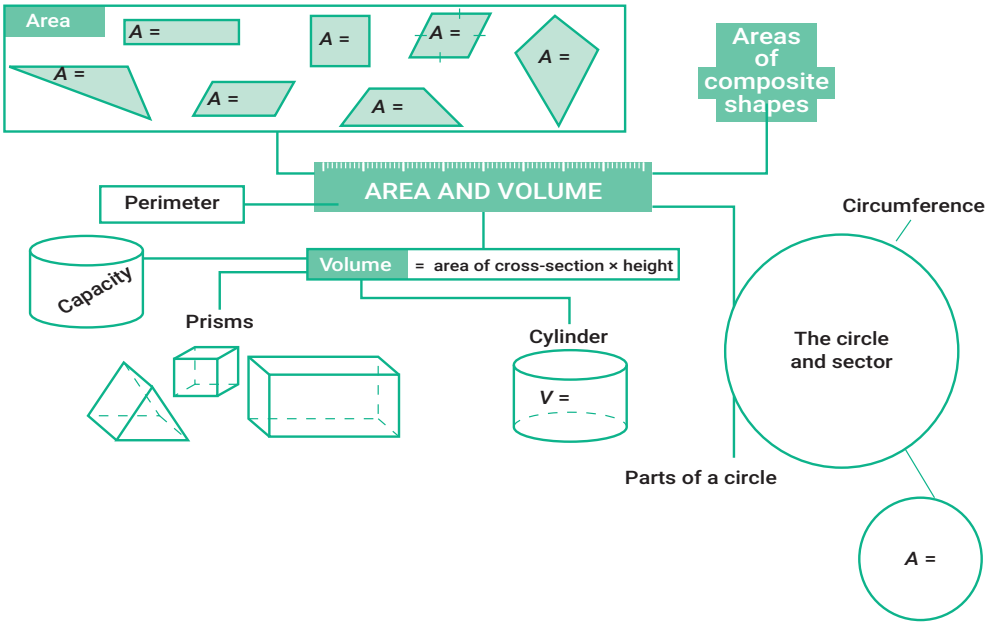
6 What shape could be called a 'circular prism'?

Topic summary

- What new things did you learn in this chapter?
- How often do you measure items? Give examples of the types of objects measured in your home.
- List 4 occupations that rely on being able to measure accurately.
- Did you have problems with any of the questions in this topic? If you did, discuss them with your teacher or a friend.

Mind map: Area and volume

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 5

ANSWERS ON P. 559

5.01

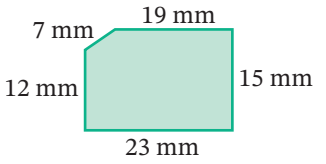
1 What is the perimeter of a rectangle with length 8 m and width 4 m?
Select the correct answer **A**, **B**, **C** or **D**.

A 12 m **B** 16 m **C** 24 m **D** 48 m

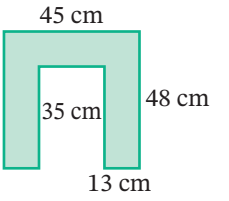
5.01

2 Find the perimeter of each shape.

a



b



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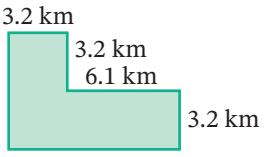


5.02

3 Copy and complete each conversion.

a 200 mm^2 = _____ cm^2 **b** 7 m^2 = _____ cm^2 **c** 8000 cm^2 = _____ m^2

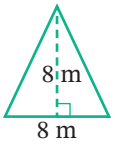
d 30 km^2 = _____ m^2 **e** 70 000 cm^2 = _____ m^2 **f** 770 mm^2 = _____ cm^2

g 700 mm^2 = _____ m^2 **h** 7.2 ha = _____ m^2 **i** 5.8 mm^2 = _____ cm^2

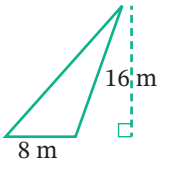
5.03

4 Which triangle has an area of 64 square metres?

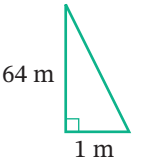
A



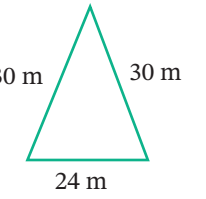
B



C



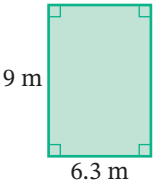
D



5.03

5 Find the area of each shape.

a



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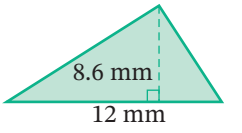

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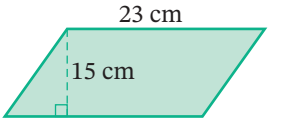
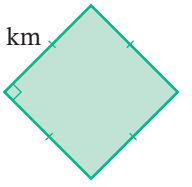


d



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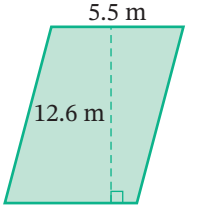

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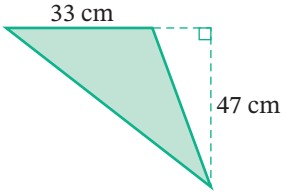

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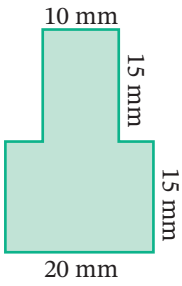
6 Find the area of each shape. 5.04

a



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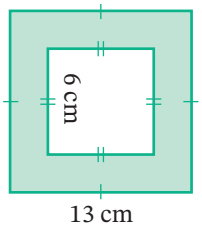

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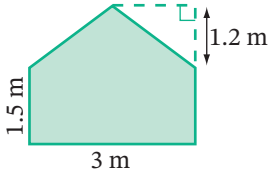

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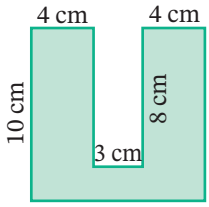
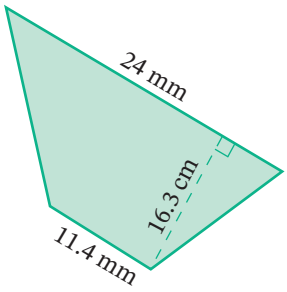


7 Find the area of each shape. 5.05

a



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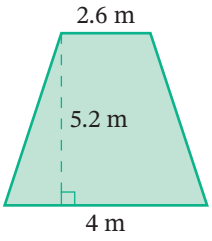

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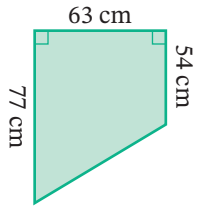
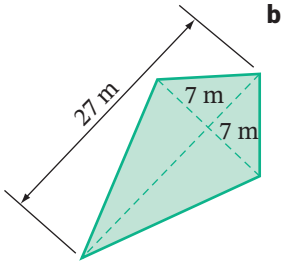


8 Find the area of each shape. 5.06

a



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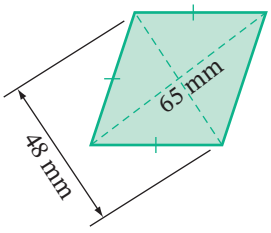

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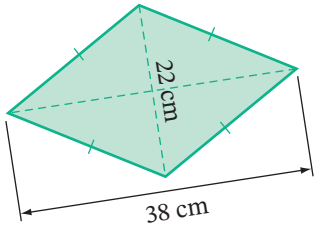
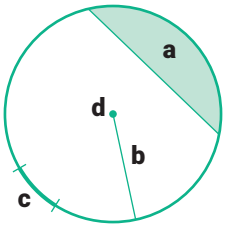
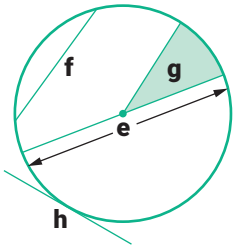


9 Name each part of the circle labelled. 5.07





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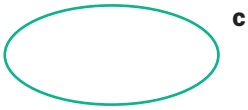
5.07

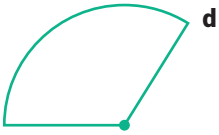
10 For each shape, state:

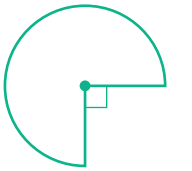
i the number of axes of symmetry

ii the order of rotational symmetry if it has rotational symmetry

a 

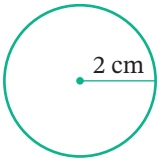
b 

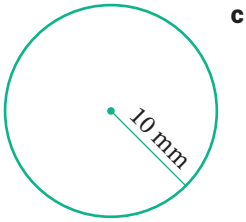
c 

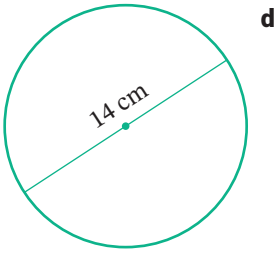
d 

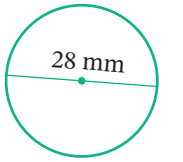
5.08

11 Calculate, correct to one decimal place, the circumference of each circle.

a 

b 

c 

d 

5.09

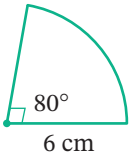
12 Calculate the area of each circle above:

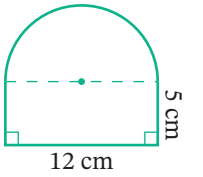
i in terms of π

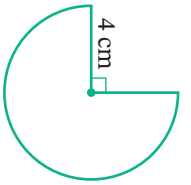
ii correct to one decimal place

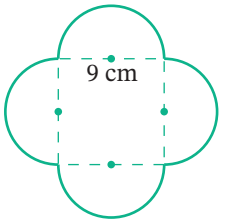
5.10

13 Calculate the perimeter of each shape, correct to 2 decimal places.

a 

b 

c 

d 

5.10

14 Calculate, correct to 2 decimal places, the area of each shape above.

5.11

15 Copy and complete each conversion.

a $8.2\ m^3 =$ _____ cm^3

b $3.4\ cm^3 =$ _____ mm^3

c $2\,000\,000\ cm^3 =$ _____ m^3

d $1\ km^3 =$ _____ m^3

e $8\ m^3 =$ _____ mm^3

f $45\,000\ mm^3 =$ _____ cm^3

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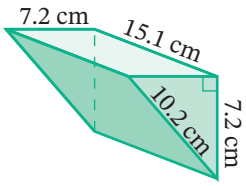
5.12

16 Find the volume of each prism.

a



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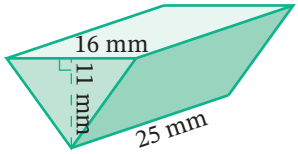

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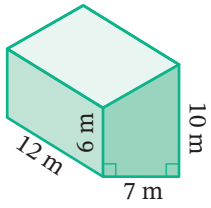

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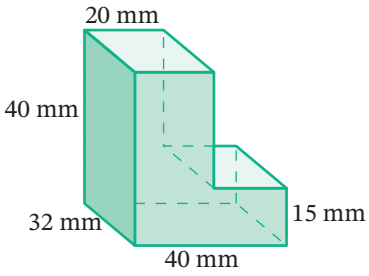
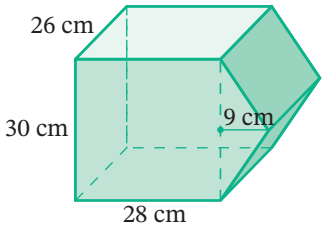


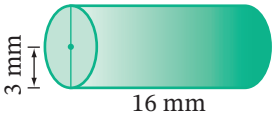
e



17 Find the volume of each solid, correct to 2 decimal places.

5.13

a



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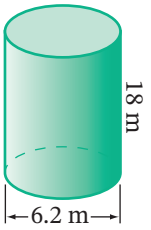

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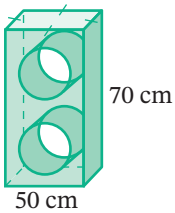


Diameter of circles = 16 cm

18 Copy and complete each conversion.

5.14

a 3 L = _____mL

b 2500 mL = _____L

c 6.5 kL = _____L

d 7.2 mm^3 = _____mL

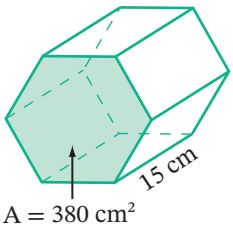
e 120 mL = _____L

f 35 cm^3 = _____L

19 Find the capacity of each solid in litres (correct to one decimal place for **c**).

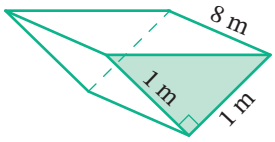
5.14

a



A = 380 cm^2

b



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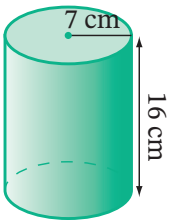


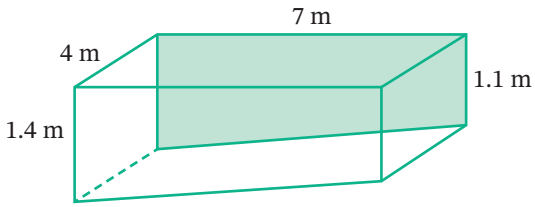
20 Shreya's pool has the shape of a trapezoidal prism.
The shaded area is a trapezium.

5.14

a Find the shaded area.

b Find the volume of the pool.

c Find the capacity of the pool in litres.



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